PART –A (PHYSICS)

1. At a given instant, say t = 0, two radioactive substances A and B have equal activates. The ratio $\frac{R_B}{R_A}$ of their activities. The ratio $\frac{R_B}{R_A}$ of their activates after time t itself decays with time t as e^{-3t}. If the half-life of A is $\ell n2$, the half-life of B is:

(A) $4\ell n2$
(B) $\frac{\ell n2}{2}$
(C) $\frac{\ell n2}{4}$
(D) $2\ell n2$

2. A power transmission line feeds input power at 2300 V to a step down transformer with its primary windings having 4000 turns. The output power is delivered at 230 V by the transformer. If the current in the primary of the transformer is 5A and its efficiency is 90%, the output current would be:

(A) 50 A
(B) 45 A
(C) 35 A
(D) 25 A

3. The energy associated with electric field is (U_E) and with magnetic field is (U_B) for an electromagnetic wave in free space. Then:

(A) $U_E = \frac{U_B}{2}$
(B) $U_E > U_B$
(C) $U_E < U_B$
(D) $U_E = U_B$

4. A force acts on a 2 kg object so that its position is given as a function of time as $x = 3t^2 + 5$. What is the work done by this force in first 5 seconds?

(A) 850 J
(B) 950 J
(C) 875 J
(D) 900 J

5. A particle having the same charge as of electron moves in a circular path of radius 0.5 cm under the influence of a magnetic field of 0.5 T. If an electric field of 100 V/m makes it to move in a straight path, then the mass of the particle is (given charge of electron $= 1.6 \times 10^{-19}$ C)

(A) 9.1×10^{-31} kg
(B) 1.6×10^{-27} kg
(C) 1.6×10^{-19} kg
(D) 2.0×10^{-24} kg

6. Two point charges $q_1(\sqrt{10}\ \mu C)$ and $q_2(-25\ \mu C)$ are placed on the x-axis at x = 1 m and x = 4 m respectively. The electric field (in V/m) at a point y = 3 m on y-axis is, $\left[\text{take}\ \frac{1}{4\pi\varepsilon_0} = 9 \times 10^9\ Nm^2C^{-2}\right]$

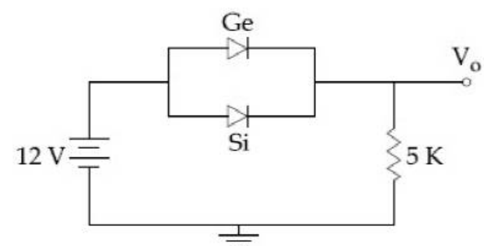
(A) $(63\hat{i} - 27\hat{i}) \times 10^2$
(B) $(-63\hat{i} + 27\hat{i}) \times 10^2$
(C) $(81\hat{i} - 81\hat{i}) \times 10^2$
(D) $(-81\hat{i} + 81\hat{i}) \times 10^2$

7. Expression for time in terms of G (universal gravitational constant), h (Planck constant) and c (speed of light) is proportional to:

(A) $\sqrt{\frac{hc^5}{G}}$ (B) $\sqrt{\frac{c^3}{Gh}}$

(C) $\sqrt{\frac{Gh}{c^5}}$ (D) $\sqrt{\frac{Gh}{c^3}}$

8. Ge and Si diodes start conducting at 0.3 V and 0.7 V respectively. In the following figure if Ge diode connection are reversed, the value of V_o changes by: (assume that the Ge diode has large breakdown voltage)



(A) 0.8 V
(B) 0.6 V
(C) 0.2 V
(D) 0.4 V

9. The top of a water tank is open to air and its water level is maintained. It is giving out 0.74 m^3 water per minute through a circular opening of 2 cm radius in its wall. The depth of the centre of the opening from the level of water in the tank is close to:

(A) 6.0 m (B) 4.8 m
(C) 9.6 m (D) 2.9 m

10. The energy required to take a satellite to a height 'h' above Earth surface (radius of Earth = 6.4×10^3 km) is E_1 and kinetic energy required for the satellite to be in a circular orbit at this height is E_2. The value of h for which E_1 and E_2 are equal is

(A) 1.6×10^3 km (B) 3.2×10^3 km
(C) 6.4×10^3 km (D) 1.28×10^4 km

11. Two Carnot engines A and B are operated in series. The first one, A, receives heat at T_1(= 600 K) and rejects to a reservoir at temperature T_2. The second engine B receives heat rejected by the first engine and, in turns, rejects to a heat reservoir at T_3 (=400 K). Calculate the temperature T_2 if the work outputs of the two engines are equal:

(A) 600 K (B) 400 K
(C) 300 K (D) 500 K

12. A series AC circuit containing an inductor (20 mH), a capacitor (120 μF) and a resistor (60 Ω) is driven by an AC source of 24 V/50 Hz. The energy dissipated in the circuit in 60 s is:

(A) 5.65×10^2 J (B) 2.26×10^3 J
(C) 5.17×10^2 J (D) 3.39×10^3 J

13. A particle is executing simple harmonic motion (SHM) of amplitude A, along the x-axis, about x = 0. When its potential energy (PE) equals kinetic energy (KE), the position of the particle will be

(A) $\frac{A}{2}$ (B) $\frac{A}{2\sqrt{2}}$

(C) $\frac{A}{\sqrt{2}}$ (D) A

14. A mass of 10 kg is suspended vertically by a rope from the roof. When a horizontal force is applied on the rope at some point, the rope deviated at an angle of 45° at the roof point. If the suspended mass is at equilibrium, the magnitude of the force applied is (g = 10 ms^{-2})

(A) 200 N (B) 140 N
(C) 70 N (D) 100 N

15. A 15 g mass of nitrogen gas is enclosed in a vessel at a temperature 27°C. Amount of heat transferred to the gas, so that rms velocity of molecules is doubled, is about: [Take R = 8.3 J/K mole]

(A) 0.9 kJ (B) 6 kJ
(C) 10 kJ (D) 14 kJ

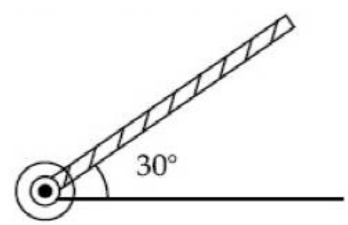
16. In a Young's double slit experiment, the slits are placed 0.320 mm apart. Light of wavelength λ = 500 nm is incident on the slits. The total number of bright fringes that are observed in the angular range $-30° \le \theta \le 30°$ is:

(A) 640 (B) 320
(C) 321 (D) 641

17. Two plane mirrors are inclined to each other such that a ray of light incident to the first mirror (M_1) and parallel to the second mirror (M_2) is finally reflected from the second mirror (M_2) parallel to the first mirror (M_1). The angle between the two mirrors will be:

(A) 45° (B) 60°
(C) 75° (D) 90°

18. A rod of length 50 cm is pivoted at one end. It is raised such that if makes an angle of 30° fro the horizontal as shown and released from rest. Its angular speed when it passes through the horizontal (in rad s^{-1}) will be (g = 10 ms^{-2}).



(A) $\sqrt{\frac{30}{2}}$ (B) $\sqrt{30}$

(C) $\sqrt{\frac{20}{2}}$ (D) $\frac{\sqrt{30}}{2}$

19. A carbon resistance has a following colour code. What is the value of the resistance?

(A) 530 k$\Omega \pm$ 5% (B) 5.3 M$\Omega \pm$ 5%
(C) 6.4 M$\Omega \pm$ 5% (D) 64 k$\Omega \pm$ 10%

20. One of the two identical conducing wires of length L is bent in the form of a circular loop and the other one into a circular coil of N identical turns. If the same current is passed in both, the ratio of the magnetic field at the central of the loop (B_L) to that at the centre of the coil (B_C), i.e. $\frac{B_L}{B_C}$ will be

(A) N (B) $\frac{1}{N}$

(C) N^2 (D) $\frac{1}{N^2}$

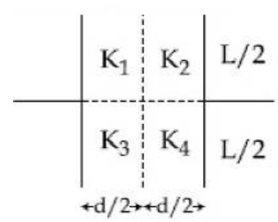
21. A rod of mass 'M' and length '2L' is suspended at its middle by a wire. It exhibits torsional oscillations; If two masses each of 'm' are attached at distance 'L/2' from its centre on both sides, it reduces the oscillation frequency by 20%. The value of ratio m/M is close to:
(A) 0.77 (B) 0.57
(C) 0.37 (D) 0.17

22. Charge is distributed within a sphere of radius R with a volume charge density $\rho(r) = \frac{A}{r^2} e^{-2r/a}$, where A and a are constants. If Q is the total charge of this charge distribution, the radius R is:

(A) $a \log\left(1 - \frac{Q}{2\pi a A}\right)$ (B) $\frac{a}{2} \log\left(\frac{1}{1 - \frac{Q}{2\pi a A}}\right)$

(C) $\frac{a}{2} \log\left(\frac{1}{1 - \frac{Q}{2\pi a A}}\right)$ (D) $\frac{a}{2} \log\left(1 - \frac{1}{2\pi a A}\right)$

23. A parallel palate capacitor with square plates is filled with four dielectrics of dielectric constants K_1, K_2, K_3, K_4 arranged as shown in the figure. The effective dielectric constant K will be:



(A) $K = \frac{(K_1 + K_3)(K_2 + K_4)}{K_1 + K_2 + K_3 + K_4}$ (B) $K = \frac{(K_1 + K_2)(K_3 + K_4)}{2(K_1 + K_2 + K_3 + K_4)}$

(C) $K = \frac{(K_1 + K_2)(K_3 + K_4)}{K_1 + K_2 + K_3 + K_4}$ (D) $K = \frac{(K_1 + K_4)(K_2 + K_3)}{2(K_1 + K_2 + K_3 + K_4)}$

24. The pitch and the number of divisions, on the circular scale, for a given screw gauge are 0.5 mm and 100 respectively. When the screw gauge is fully tightened without any object, the zero of its circular scale lies 3 divisions below the mean line.
The readings of the main scale and the circular scale for a thin sheet, are 5.5 mm and 48 respectively, the thickness of this sheet is
(A) 5.755 mm (B) 5.950 mm
(C) 5.725 mm (D) 5.740 mm

25. A musician using an open flute of length 50 cm produces second harmonic sound waves.
A person runs towards the musician from another end of a hall at a speed of 10 km/h. If the wave speed is 330 m/s, the frequency heard by the running person shall be close to:
(A) 666 Hz (B) 753 Hz
(C) 500 Hz (D) 333 Hz

26. In a car race on straight road, car A takes a time 't' less than car B at the finish and passes finishing point with a speed 'v' more than that of car B. Both the cars start from rest and travel with constant acceleration a_1 and a_2 respectively. Then 'v' is equal to

(A) $\frac{2a_1a_1}{a_1 + a_2}t$ (B) $\sqrt{2a_1a_2}\ t$

(C) $\sqrt{a_1a_2}\ t$ (D) $\frac{a_1 + a_2}{2}t$

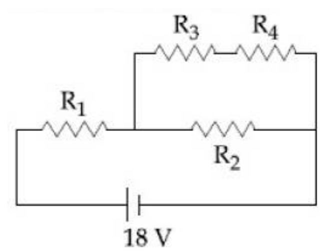
27. The magnetic field associated with a light wave is given, at the origin, by

$$B = B_0\,[\sin(3.14 \times 10^7)ct + \sin(6.28 \times 10^7)ct]$$

If this light falls on a silver plate having a work function of 4.7 eV, what will be the maximum kinetic energy of the photo electrons?

(A) 6.82 eV (B) 12.5 eV
(C) 8.52 eV (D) 7.72 eV

28. In the given circuit the internal resistance of the 18 V cell is negligible. If $R_1 = 400\ \Omega$, $R_3 = 100\ \Omega$ and $R_4 = 500\ \Omega$ and the reading of an ideal voltmeter across R_4 is 5 V, then the value of R_2 will be



(A) 300 Ω (B) 450 Ω
(C) 550 Ω (D) 230 Ω

29. In a communication system operating at wavelength 800 nm, only one percent of source frequency is available as signal bandwidth. The number of channels accommodated for transmitting TV signals of band width 6 MHz are (Take velocity of light $c = 3 \times 10^8$ m/s, $h = 6.6 \times 10^{-34}$ J-s)

(A) 3.75×10^6 (B) 3.86×10^6
(C) 6.25×10^5 (D) 4.87×10^5

30. The position co-ordinates of a particle moving in a 3-D coordinates system is given by

$x = a\cos\omega t$

$y = a\sin\omega t$

and $z = a\omega t$

The speed of the particle is:

(A) $\sqrt{2}\ a\omega$ (B) $a\omega$

(C) $\sqrt{3}\ a\omega$ (D) $2a\omega$

PART –B (CHEMISTRY)

31. The entropy change associated with the conversion of 1 kg of ice at 273 K to water vapours at 383 K is: (Specific heat of water liquid and water vapours are 4.2 kJ K^{-1} kg^{-1} and 2.0 kJ K^{-1} kg^{-1}, heat of liquid fusion and vapourisation of water are 334 kJ kg^{-1} and 2491 kJ kg^{-1}, respectively) (log 273 = 2.436, log 373 = 2.572, log 383 = 2.583)

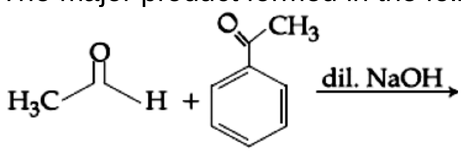
(A) 7.90 kJ K^{-1} kg^{-1}
(B) 2.64 kJ K^{-1} kg^{-1}
(C) 8.49 kJ K^{-1} kg^{-1}
(D) 9.26 kJ K^{-1} kg^{-1}

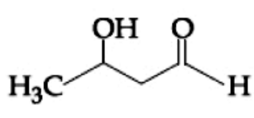
32. For the following reaction the mass of water produced from 445 g of $C_{57}H_{110}O_6$ is

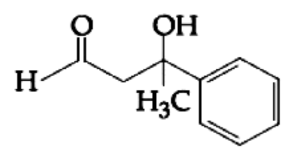
$$2C_{57}H_{110}O_6(s) + 163O_2(g) \longrightarrow 114CO_2(g) + 110H_2O(l)$$

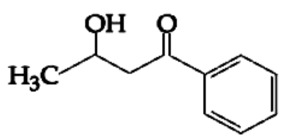
(A) 490 g
(B) 445 g
(C) 495 g
(D) 890 g

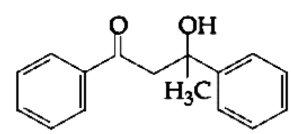
33. The major product formed in the following reaction is:



(A) 

(B) 

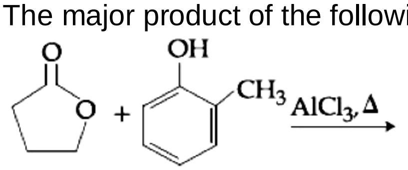
(C) 

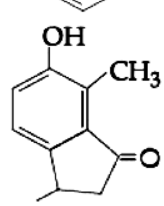
(D) 

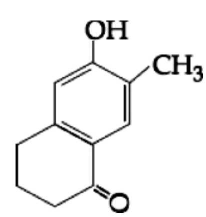
34. Which of the following conditions in drinking water causes methemoglobinemia?

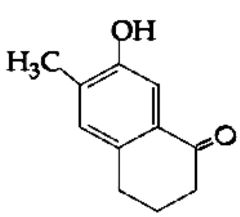
(A) > 50 ppm of lead
(B) > 50 ppm of chloride
(C) > 50 ppm of nitrate
(D) > 100 ppm of sulphate

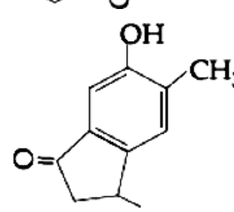
35. The major product of the following reaction is:



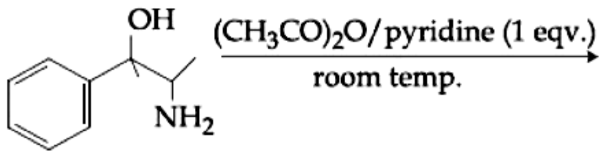
(A) 

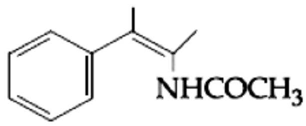
(B) 

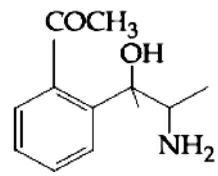
(C) 

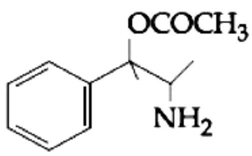
(D) 

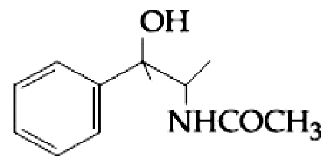
36. The major product obtained in the following reaction is:



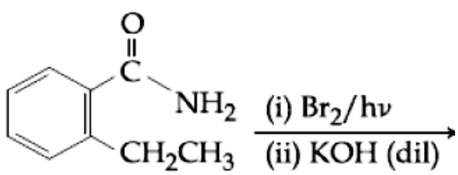
(A) 

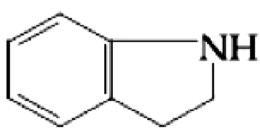
(B) 

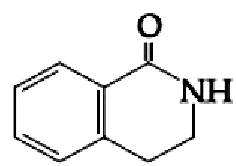
(C) 

(D) 

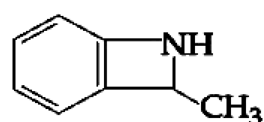
37. The major product of the following reaction is:



(A) 

(B) 

(C) 

(D) 

38. The correct match between item I and item II is

	Item I		**Item II**
(a)	Benzaldehyde	(p)	Mobile phase
(b)	Alumina	(q)	Adsorbent
(c)	Acetonitrile	(r)	Adsorbate

(A) a $\rightarrow$ q, b $\rightarrow$ p, c $\rightarrow$ r
(B) a $\rightarrow$ r, b $\rightarrow$ q, c $\rightarrow$ p
(C) a $\rightarrow$ q, b $\rightarrow$ r, c $\rightarrow$ p
(D) a $\rightarrow$ p, b $\rightarrow$ r, c $\rightarrow$ q

39. The metal that forms nitride by reacting directly with N_2 of air is
(A) K
(B) Li
(C) Rb
(D) Cs

40. For coagulation of arsenious sulphide sol, which one of the following salt solution will be most effective?
(A) $BaCl_2$
(B) $AlCl_3$
(C) NaCl
(D) Na_3PO_4

41. The complex that has highest crystal field splitting energy(Δ) is
(A) $[Co(NH_3)_5(H_2O)]Cl_3$
(B) $K_2[CoCl_4]$
(C) $[Co(NH_3)_5Cl]Cl_2$
(D) $K_3[Co(CN)_6]$

42. The pH of rain water is approximately
(A) 5.6
(B) 7.5
(C) 7.0
(D) 6.5

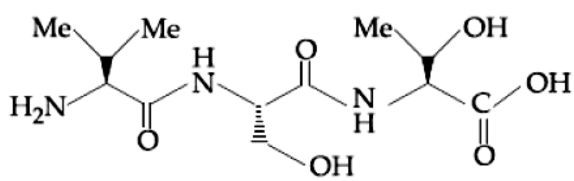
43. Consider the following reversible chemical reactions:

$$A_2(g) + B_2(g) \underset{}{\overset{K_1}{\rightleftharpoons}} 2\,AB(g) \quad ....(1)$$

$$6\,AB(g) \underset{}{\overset{K_2}{\rightleftharpoons}} 3\,A_2(g) + 3\,B_2(g) \quad ....(2)$$

The relation between K_1 and K_2 is
(A) $K_1K_2 = \frac{1}{3}$
(B) $K_2 = K_1^3$
(C) $K_2 = K_1^{-3}$
(D) $K_1K_2 = 3$

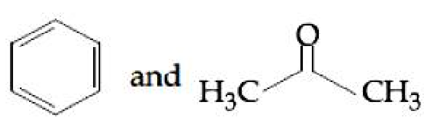
44. The correct sequence of amino acids present in the tripeptide given below is

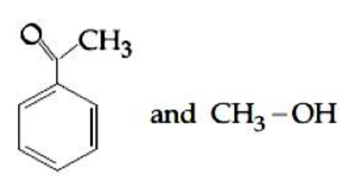


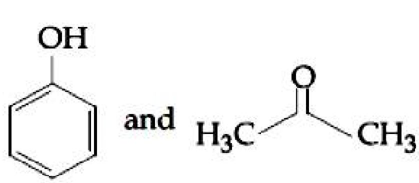
(A) Val – Ser – Thr
(B) Thr – Ser – Val
(C) Leu – Ser – Thr
(D) Thr – Ser – Leu

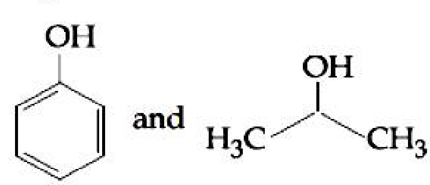
45. For the reaction, $2A + B \longrightarrow \text{products}$, when the concentrations of A and B both were doubled, the rate of the reaction increased from 0.3 mol L^{-1} s^{-1} to 2.4 mol L^{-1} s^{-1}. When the concentration of A alone is doubled, the rate increased from 0.3 mol L^{-1} s^{-1} to 0.6 mol L^{-1} s^{-1}.
Which one of the following statements is correct?
(A) Total order of the reaction is 4
(B) Order of the reaction with respect to B is 2
(C) Order of the reaction with respect to B is 1
(D) Order of the reaction with respect to A is 2

46. The products formed in the reaction of cumene with O_2 followed by treatment with dil. HCl are:

(A) 

(B) 

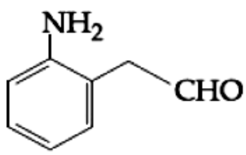
(C) 

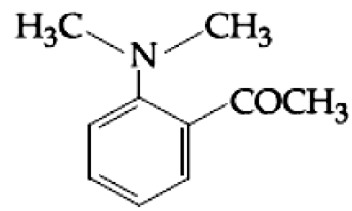
(D) 

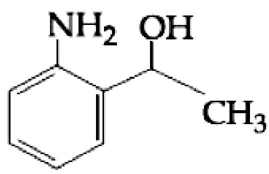
47. The tests performed on compound X and their inferences are:

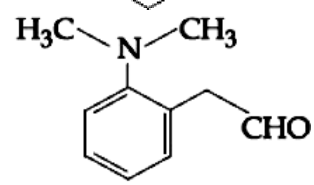
	Test	Interference
(a)	2, 4-DNP test	Coloured precipitate
(b)	Iodoform test	Yellow precipitate
(c)	Azo-dye test	No dye formation

Compound 'X' is

(A) 

(B) 

(C) 

(D) 

48. If the standard electrode potential for a cell is 2 V at 300 K, the equilibrium constant (K) for the reaction

$Zn(s) + Cu^{2+}(aq) \rightleftharpoons Zn^{2+}(aq) + Cu(s)$

At 300 K is approximately

(R = 8 JK^{-1} mol^{-1}, F = 96000 C mol^{-1})

(A) e^{-80} (B) e^{-160}
(C) e^{320} (D) e^{160}

49. The temporary hardness of water is due to

(A) Na_2SO_4 (B) NaCl
(C) $Ca(HCO_3)_2$ (D) $CaCl_2$

50. In which of the following processes, the bond order has increased and paramagnetic character has changed to diamagnetic?

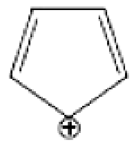
(A) $NO \longrightarrow NO^+$ (B) $N_2 \longrightarrow N_2^+$
(C) $O_2 \longrightarrow O_2^+$ (D) $O_2 \longrightarrow O_2^{2-}$

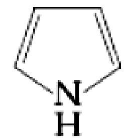
51. Which of the following combination of statements is true regarding the interpretation of the atomic orbitals?

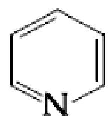
(1) An electron in an orbital of high angular momentum stays away from the nucleus than an electron in the orbital of lower angular momentum.
(2) For a given value of the principal quantum number, the size of the orbit is inversely proportional to the azimuthal quantum number
(3) According to wave mechanics, the ground state angular momentum is equal to $\frac{h}{2\pi}$
(4) The plot of ψ Vs r for various azimuthal quantum numbers, shows peak shifting towards higher r value

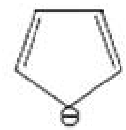
(A) (1), (4) (B) (1), (2)
(C) (1), (3) (D) (2), (3)

52. Which of the following compounds is not aromatic?

(A) 

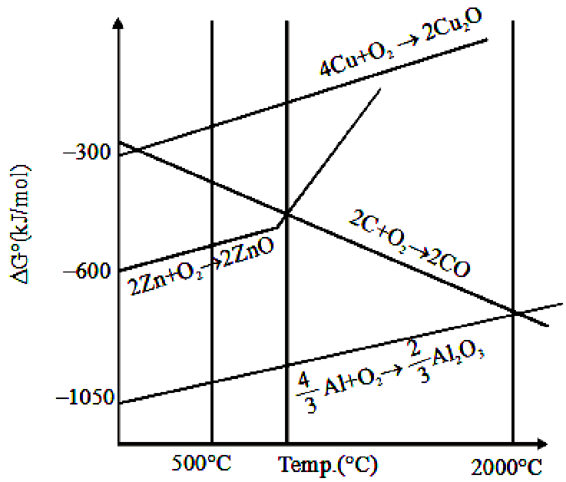
(B) 

(C) 

(D) 

53. Good reducing nature of H_3PO_2 is attributed to the presence of
(A) Two P – OH bonds
(B) One P – H bond
(C) Two P – H bonds
(D) One P – OH bond

54. The correct statement regarding the given Ellingham diagram is

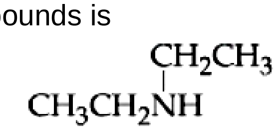


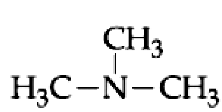
(A) At 1400°C, Al can be used for the extraction of Zn from ZnO
(B) At 500°C, coke can be used for the extraction of Zn from ZnO
(C) Coke cannot be used for the extraction of Cu from Cu_2O
(D) At 800°C Cu can be used for the extraction of Zn from ZnO

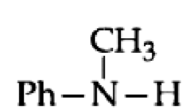
55. The transition element that has lowest enthalpy of atomisation is
(A) Fe
(B) Cu
(C) V
(D) Zn

56. The increasing basicity order of the following compounds is

(1) $CH_3CH_2NH_2$

(2) 

(3) 

(4) 

(A) (4) < (3) < (2) < (1)
(B) (4) < (3) < (1) < (2)
(C) (1) < (2) < (3) < (4)
(D) (1) < (2) < (4) < (3)

57. When the first electron gain enthalpy ($\Delta_{eg}H$) of oxygen is -141 kJ/mol, its second electron gain enthalpy is

(A) a more negative value than the first
(B) almost the same as that of the first
(C) negative, but less negative than the first
(D) a positive value

58. At 100°C, copper(Cu) has FCC unit cell structure with cell edge length $x\,\mathring{A}$. What is the approximate density of Cu(in g cm^{-3}) at this temperature?
[Atomic mass of Cu = 63.55 u]

(A) $\frac{205}{x^3}$
(B) $\frac{105}{x^3}$
(C) $\frac{211}{x^3}$
(D) $\frac{422}{x^3}$

59. A solution containing 62 g ethylene glycol in 250 g water is cooled to -10°C. If K_f for water is 1.86 K kg mol^{-1}, the amount of water(in g) separated as ice is

(A) 48
(B) 32
(C) 64
(D) 16

60. Homoleptic octahedral complexes of a metal ion 'M^{3+}' with three monodentate ligands L_1, L_2 and L_3 absorb wavelengths in the region of green, blue and red respectively. The increasing order of the ligand strength is

(A) $L_3 < L_1 < L_2$
(B) $L_3 < L_2 < L_1$
(C) $L_1 < L_2 < L_3$
(D) $L_2 < L_1 < L_3$

PART–C (MATHEMATICS)

61. The sum of the following series

$$1+6+\frac{9\left(1^2+2^2+3^2\right)}{7}+\frac{12\left(1^2+2^2+3^2+4^2\right)}{9}+\frac{15\left(1^2+2^2+....+5^2\right)}{11}+...\text{up to 15 terms, is:}$$

(A) 7820
(B) 7830
(C) 7520
(D) 7510

62. For each $x \in R$, let $[x]$ be the greatest integer less than or equal to x. Then $\lim_{x\to 0^+}\frac{x\left([x]+|x|\right)\sin[x]}{|x|}$ is equal to

(A) $-\sin 1$
(B) 0
(C) 1
(D) sin 1

63. Let $f:[0,1]\to R$ be such that $f(xy)=f(x)\,f(y)$ for all $x,y\in[0,1]$, and $f(0)\neq 0$. If $y=y(x)$ satisfies the differential equation, $\frac{dy}{dx}=f(x)$ with $y(0)=1$, then $y\left(\frac{1}{4}\right)+y\left(\frac{3}{4}\right)$ is equal to

(A) 4
(B) 3
(C) 5
(D) 2

64. If $x=\sin^{-1}(\sin 10)$ and $y=\cos^{-1}(\cos 10)$, then y – x is equal to:

(A) π
(B) 7π
(C) 0
(D) 10

65. If $0\le x<\frac{\pi}{2}$, then the number of values of x for which sin x – sin 2x + sin 3x = 0, is

(A) 2
(B) 1
(C) 3
(D) 4

66. Let z_0 be a root of the quadratic equation, $x^2+x+1=0$. If $z=3+6iz_0^{81}-3iz_0^{93}$, then $\arg z$ is equal to

(A) $\frac{\pi}{4}$
(B) $\frac{\pi}{3}$
(C) 0
(D) $\frac{\pi}{6}$

67. The area of the region $A\left\{(x,y):0\le y\le x|x|+1 \text{ and } -1\le x\le 1\right\}$ in sq. units, is:

(A) $\frac{2}{3}$
(B) $\frac{1}{3}$
(C) 2
(D) $\frac{4}{3}$

68. If the system of linear equation $x-4y+7z=g, 3y-5z=h, -2x+5y-9z=k$ is consistent, then:

(A) $g+h+k=0$
(B) $2g+h+k=0$
(C) $g+h+2k=0$
(D) $g+2h+k=0$

69. The coefficient of t^4 in the expansion of $\left(\frac{1-t^6}{1-t}\right)^3$ is

(A) 12
(B) 15
(C) 10
(D) 14

70. If both the roots of the quadratic equation $x^2-5x+4=0$ are real and distinct and they lie in the interval $[1, 5]$, then m lies in the interval.

(A) (4, 5)
(B) (3, 4)
(C) (5, 6)
(D) (–5, –4)

71. Let S be the set of all triangle in the xy – plane, each having one vertex at the origin and the other two vertices lie on coordinate axes with integral coordinates. If each triangle in S has area 50 sq. units, then the number of elements in the set S is:

(A) 9
(B) 18
(C) 32
(D) 36

72. Let a, b and c be the 7th, 11th and 13th terms respectively of a non – constant A.P. If these are also the three consecutive terms of a G.P. then $\frac{a}{c}$ is equal to:

(A) $\frac{1}{2}$
(B) 4
(C) 2
(D) $\frac{7}{13}$

73. The logical statement $\left[\sim(\sim p \vee q) \vee (p \wedge r) \wedge (\sim q \wedge r)\right]$ is equivalent to:

(A) $(p \wedge r) \wedge \sim q$
(B) $(\sim p \wedge \sim q) \wedge r$
(C) $\sim p \vee r$
(D) $(p \wedge \sim q) \vee r$

74. The equation of the plane containing the straight line $\frac{x}{2}=\frac{y}{3}=\frac{z}{4}$ and perpendicular to the plane containing the straight lines $\frac{x}{3}=\frac{y}{4}=\frac{z}{2}$ and $\frac{x}{4}=\frac{y}{2}=\frac{z}{3}$ is:

(A) $x+2y-2z=0$
(B) $x-2y+z=0$
(C) $5x+2y-4z=0$
(D) $3x+2y-3z=0$

75. A data consists of n observations:

$x_1, x_2, ..., x_n$. If $\sum_{i=1}^{n}(x_i+1)^2=9n$ and $\sum_{i=1}^{n}(x_i-1)^2=5n$, then the standard deviation of this data is:

(A) 5
(B) $\sqrt{5}$
(C) $\sqrt{7}$
(D) 2

76. If $A=\begin{bmatrix} e^t & e^{-t}\cos t & e^{-t}\sin t \\ e^t & -e^{-t}\cos t - e^{-t}\sin t & -e^{-t}\sin t + e^{-t}\cos t \\ e^t & 2e^{-t}\sin t & -2e^{-t}\cos t \end{bmatrix}$ Then A is

(A) Invertible only if $t=\frac{\pi}{2}$
(B) not invertible for any $t \in R$
(C) invertible for all $t \in R$
(D) invertible only if $t=\pi$

77. If $f(x)=\int \frac{5x^8+7x^6}{\left(x^2+1+2x^7\right)^2}dx, (x \geq 0)$ and $f(0)=0$, then the value of $f(1)$ is:

(A) $-\frac{1}{2}$
(B) $\frac{1}{2}$
(C) $-\frac{1}{4}$
(D) $\frac{1}{4}$

78. Let f be a differentiable function R to R such that $\left|f(x)-f(y)\right| \leq 2\left|x-y\right|^{\frac{3}{2}}$, for all $x, y \in R$. If $f(0)=1$ then $\int_0^1 f^2(x)dx$ is equal to

(A) 0
(B) $\frac{1}{2}$
(C) 2
(D) 1

79. If $x=3\tan t$ and $y=3\sec t$, then the value of $\frac{d^2y}{dx^2}$ at $t=\frac{\pi}{4}$, is:

(A) $\frac{3}{2\sqrt{2}}$
(B) $\frac{1}{3\sqrt{2}}$
(C) $\frac{1}{6}$
(D) $\frac{1}{6\sqrt{2}}$

80. The number of natural numbers less than 7,000 which can be formed by using the digits 0, 1, 3, 7, 9 (repetition of digits allowed) is equal to:

(A) 250
(B) 374
(C) 372
(D) 375

81. If the circles $x^2+y^2-16x-20y+164=r^2$ and $(x-4)^2+(y-7)^2=36$ intersect at two distinct points, then:

(A) $0<r<1$
(B) $1<r<11$
(C) $r>11$
(D) $r=11$

82. A hyperbola has its centre at the origin, passes through the point (4, 2) and has transverse axis of length 4 along the x – axis. Then the eccentricity of the hyperbola is:

(A) $\frac{2}{\sqrt{3}}$
(B) $\frac{3}{2}$
(C) $\sqrt{3}$
(D) 2

83. Let $A(4, -4)$ and $B(9, 6)$ be points on the parabola $y^2 = 4x$. Let C be chosen on the arc AOB of the parabola, where O is the origin, such that the area of ΔACB is maximum. Then, the area (in sq. units) of ΔACB, is:

(A) $31\frac{3}{4}$
(B) 32
(C) $30\frac{1}{2}$
(D) $31\frac{1}{4}$

84. Let the equation of two sides of a triangle be $3x - 2y + 6 = 0$ and $4x + 5y - 20 = 0$. If the orthocentre of this triangle is at (1, 1), then the equation of its third side is:

(A) $122y - 26x - 1675 = 0$
(B) $26x + 61y + 1675 = 0$
(C) $122y + 26x + 1675 = 0$
(D) $26x - 122y - 1675 = 0$

85. An urn contains 5 red and 2 green balls. A ball is drawn at random from the urn. If the drawn ball is green, then a red ball is added to the urn and if the drawn ball is red, then a green ball is added to the urn; the original ball is not returned to the urn. Now, a second ball is drawn at random from it. The probability that the second ball is red, is:

(A) $\frac{26}{49}$
(B) $\frac{32}{49}$
(C) $\frac{27}{49}$
(D) $\frac{21}{49}$

86. If the lines $x = ay + b, z = cy + d$ and $x = a'z + b', y = c'z + d'$ are perpendicular, then:

(A) $cc' + a + a' = 0$
(B) $aa' + c + c' = 0$
(C) $ab' + bc' + 1 = 0$
(D) $bb' + cc' + 1 = 0$

87. Let $\vec{a} = \hat{i} + \hat{j} + \sqrt{2}\hat{k}, \vec{b} = b_1\hat{i} + b_2\hat{j} + \sqrt{2}\hat{k}$ and $\vec{c} = 5\hat{i} + \hat{j} + \sqrt{2}\hat{k}$ be three vectors such that the projection vector of $\vec{b}$ on $\vec{a}$ is $\vec{a}$. If $\vec{a} + \vec{b}$ is perpendicular to $\vec{c}$, then $|\vec{b}|$ is equal to:

(A) $\sqrt{22}$
(B) 4
(C) $\sqrt{32}$
(D) 6

88. The number of all possible positive integral values of α for which the roots of the quadratic equation, $6x^2 - 11x + \alpha = 0$ are rational numbers is:

(A) 2
(B) 5
(C) 3
(D) 4

89. Let $A = \{x \in R : x \text{ is not a positive integer}\}$ Define a function $f : A \to R$ as $f(x) = \frac{2x}{x-1}$ then f is

(A) injective but nor surjective
(B) not injective
(C) surjective but not injective
(D) neither injective nor surjective

90. If $\int_0^{\frac{\pi}{3}} \frac{\tan\theta}{\sqrt{2k\sec\theta}} d\theta = 1 - \frac{1}{\sqrt{2}}, (k > 0)$, then the value of k is:

(A) 2
(B) $\frac{1}{2}$
(C) 4
(D) 1

HINTS AND SOLUTIONS

PART A – PHYSICS

1. $R = R_o e^{-\lambda t}$

$\therefore \quad \frac{R_B}{R_A} = \frac{R_o e^{-\lambda_B t}}{R_o e^{-\lambda_B t}} = e^{-(\lambda_B - \lambda_A)t} = e^{-3t}$

$\Rightarrow \quad \lambda_B - \lambda_A = 3$

$\Rightarrow \quad \frac{\ell n^2}{T_B} - \frac{\ell n2}{\ell n2} = 3.$

$\Rightarrow \quad T_B = \frac{\ell n2}{4}$

2. $P_s = \eta P_P$

$\Rightarrow \quad E_s i_s = \eta E_i i_p$

$\Rightarrow \quad i_s = \frac{(0.9)(2300)(5)}{(230)} = 45$ A.

3. $B = \frac{E}{C}$

$\Rightarrow \quad U_E = \frac{1}{2}\varepsilon_o E^2$

$U_B = \frac{B^2}{2\mu_o} = \frac{E^2}{2\mu_o C^2} = \frac{E^2}{2\mu_o}(\mu_o \varepsilon_o) = U_E$

4. $x = 3t^2 + 5$

$\Rightarrow \quad v = 6t$

$\Rightarrow \quad \Delta W = \Delta k$

$= \frac{1}{2}(2)(30)^2 - \frac{1}{2}2(0)^2$

$= 900$ J

5. $eE = evB$

$\Rightarrow \quad E = \left(\frac{eBr}{m}\right)B$

$\Rightarrow \quad m = \frac{eB^2 r}{E}$

$\Rightarrow \quad m = \frac{(1.6\times10^{-19})(0.5)^2\,(0.5\times10^{-2})}{100} = 2\times10^{-24}$ kg.

6. $\vec{E} = \frac{kq_1}{r_1^3}\vec{r}_1 + \frac{kq_2}{r_2^3}\vec{r}_2 = k\times10^{-6}\left[\frac{\sqrt{10}}{10\sqrt{10}}(-\hat{i}+3\hat{j}) + \frac{(-25)}{125}(-4\hat{i}+3\hat{j})\right]$

$= (9\times10^3)\left[\frac{1}{10}(-\hat{i}+3\hat{j}) - \frac{1}{5}(-4\hat{i}+3\hat{j})\right]$

$$= (9\times10^3)\left[\left(-\frac{1}{10}+\frac{4}{5}\right)\hat{i}+\left(\frac{3}{10}-\frac{3}{5}\right)\hat{i}\right]=9000\left(\frac{7}{10}\hat{i}-\frac{3}{10}\hat{j}\right)$$

$$= (63\hat{i}-27\hat{j})\,(100)$$

7. $t = G^a h^b c^c$

$\Rightarrow$ $M^o L^o T' = (M^{-1} L^3 T^{-2})^a (ML^2T^{-1})^b (LT^{-1})^c$

$\Rightarrow$ $-a + b = 0 \Rightarrow a = b$

$\Rightarrow$ $3a + 2b + c = 0$

$\Rightarrow$ $c = -5a$

$\Rightarrow$ $-2a - b - c = 1$

$\Rightarrow$ $a=\frac{1}{2}\ ;\ \ b=\frac{1}{2}\ ;\ c=-\frac{5}{2}$

8. $V_{O_i} = 12 - 0.3 = 11.7\text{ V}$

$V_{O_f} = 12 - 0.7 = 11.3\text{ V}$

$\Rightarrow$ $\Delta V_o = -0.4\text{ V}$

9. $\frac{dV}{dt} = A\nu \quad \Rightarrow \frac{dV}{dt} = A\sqrt{2gh}$

$\Rightarrow$ $\frac{0.74}{60} = (3.14)\left(\frac{2}{100}\right)^2\sqrt{2(9.8)h}$

$\Rightarrow$ $h = 4.92\text{ m}$

10. $E_1 = -\frac{GMm}{R+h} - \left(-\frac{GMm}{R}\right)$

$E_2 = \frac{1}{2}m\left(\sqrt{\frac{GM}{R+h}}\right)^2 = \frac{GMm}{2(R+h)}$

$E_1 = E_2 \quad ; \quad h = \frac{R}{2}$

11. $W_1 = W_2$

$\Rightarrow$ $600 - T_2 = T_2 - 400$

$\Rightarrow$ $T_2 = 500\text{ K}$

12. $E = Pt = \frac{E^2}{Z^2}Rt = \frac{(24)^2}{60^2 + (8.33\pi - 2\pi)^2}(60)\,(60) = 518\text{ J.}$

13. PE = KE

$\Rightarrow$ $\frac{1}{2}m\omega^2(A^2 - x^2) = \frac{1}{2}m\omega^2x^2$

$\Rightarrow$ $x = \frac{A}{\sqrt{2}}$

14. T cos 45° = mg

T sin 45° = F

$\Rightarrow$ F = mg = 100 N.

15. $\Delta Q = \frac{f}{2} nR\Delta T$

$= \frac{5}{2}\left(\frac{15}{28}\right)(8.3)(1200 - 300)$ = 10000 J.

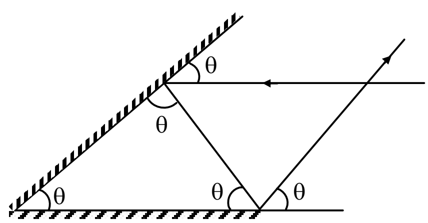
16. $\Delta X_{max} = d \sin\theta = 0.32 \sin 30 = 0.16$ mm

$\therefore \; n = \frac{\Delta X_{max}}{\lambda} = \frac{0.16 \times 10^{-3}}{500 \times 10^{-9}}$

$= \frac{0.16 \times 10^{6}}{500} = \frac{1600}{5} = 320$

$\therefore$ Number of BFs = (2n + 1) = 641

17.



$\theta = 60°$

18. $mg\frac{\ell}{2}\left(\frac{1}{2}\right) = \frac{1}{2}\left(\frac{m\ell^2}{3}\right)\omega^2$

$\Rightarrow \quad \omega = \sqrt{\frac{3g}{2\ell}} = \sqrt{30}$

19. R = 530 k$\Omega \pm 5\%$

20. $B_L = \frac{\mu_o i}{2R}$

$B_C = \frac{\mu_o N i}{2(R/N)}$

$\therefore \; \frac{B_L}{B_C} = \frac{1}{N^2}$

21. $f = \frac{1}{2\pi}\sqrt{\frac{C}{\left(\frac{ML^2}{3}\right)}}$ & $0.8 f = \frac{1}{2\pi}\sqrt{\frac{C}{\left(\frac{ML^2}{3} + \frac{mL^2}{2}\right)}}$

$\Rightarrow \quad \frac{25}{16} = \frac{\frac{ML^2}{3} + \frac{mL^2}{2}}{\frac{ML^2}{3}}$

$$\Rightarrow \quad \frac{25}{16} = 1 + \frac{3\,m}{2\,M}$$

$$\Rightarrow \quad \frac{9}{16} = \frac{3\,m}{2\,M}$$

$$\Rightarrow \quad \frac{m}{M} = \frac{3}{8} = 0.37$$

22. $$Q = \int \rho 4\pi r^2 dr = \int_0^R \left(\frac{A}{r^2} e^{-\frac{2r}{a}}\right)(4\pi r^2)\,dr$$

$$= 4\pi A \frac{a}{2}\left(1 - e^{\frac{-2R}{a}}\right)$$

$$\Rightarrow \quad R = \frac{-a}{2}\log\left(1 - \frac{Q}{2\pi Aa}\right)$$

23. $$C_1 = \frac{\varepsilon_0 K_1 \frac{L^2}{2}}{\frac{d}{2}} + \frac{\varepsilon_0 K_3 \frac{L^2}{2}}{\left(\frac{d}{2}\right)} = \frac{\varepsilon_0 L^2}{d}(K_1 + K_3)$$

$$C_2 = \frac{\varepsilon_o K_2 \frac{L^2}{2}}{\frac{d}{2}} + \frac{\varepsilon_o K_4 \frac{L^2}{2}}{\frac{d}{2}} = \frac{\varepsilon_o L^2}{d}(K_2 + K_4)$$

$$\therefore \quad \frac{1}{c} = \frac{1}{c_1} + \frac{1}{c_2}$$

$$\Rightarrow \quad \frac{d}{\varepsilon_o K L^2} = \frac{d}{\varepsilon_o L^2 (K_1 + K_3)} + \frac{d}{\varepsilon_o L^2 (K_2 + K_4)}$$

24. Zero error = $0 + 3 \times \frac{0.5\,mm}{100} = 0.015\,mm$

MSR = $5.5 + 48 \times \frac{0.5}{100}$

= 5.74 mm.

∴ Thickness = 5.74 – 0.015 = 5.725 mm

25. $$f = \frac{2}{2\ell} v_s = \frac{330}{0.5} = 660\,Hz$$

$$\therefore \quad f' = f\left(\frac{\nu_s + \nu}{\nu_s}\right) = (660)\left(\frac{330 + \frac{50}{18}}{330}\right) = 660\left(1 + \frac{50}{18 \times 330}\right)$$

= 666 Hz.

26. $\sqrt{\frac{2\ell}{a_2}} - \sqrt{\frac{2\ell}{a_1}} = t \qquad \Rightarrow \qquad \frac{\sqrt{2\ell}}{t} = \frac{\sqrt{a_1 a_2}}{\sqrt{a_1} - \sqrt{a_2}}$

$\sqrt{2a_1\ell} - \sqrt{2a_2\ell} = \nu \qquad \Rightarrow \qquad \frac{\sqrt{2\ell}}{v} = \frac{1}{\sqrt{a_1} - \sqrt{a_2}}$

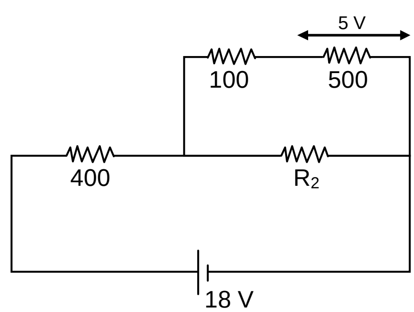
$\Rightarrow \quad \frac{\nu}{t} = \sqrt{a_1 a_2} \qquad \Rightarrow \qquad \nu = (\sqrt{a_1 a_2})\, t$

27. $KE_{max} = h\nu_{max} - \phi$

$= \frac{(6.6\times 10^{-34})(6.28\times 10^{7})(3\times 10^{8})}{1.6\times 10^{-19}\times 2\times 3.14} - 4.7$

$= 12.37 - 4.7 = 7.67$ eV

28.



$\frac{12}{400} = \frac{6}{600} + \frac{6}{R_2}$

$\Rightarrow \quad \frac{1}{200} = \frac{1}{600} + \frac{1}{R_2}$

$\Rightarrow \quad R_2 = 300\ \Omega$

29. $f = \frac{c}{\lambda} = \frac{3\times 10^{8}}{8\times 10^{-7}} = \frac{3}{8}\times 10^{15}\,\text{Hz}$

$\therefore \quad n = \frac{(0.01)\,f}{6\times 10^{6}} = \frac{\frac{3}{8}\times 10^{13}}{6\times 10^{6}}$

$= \frac{1}{16}\times 10^{7} = 6.25\times 10^{5}$

30. $v_x = \frac{dx}{dt} = -a\omega \sin\omega t$

$v_y = \frac{dy}{dt} = a\omega\cos\omega t$

$v_z = \frac{dz}{dt} = a\omega$

$\therefore \quad v = \sqrt{v_x^2 + v_y^2 + v_z^2} = a\omega\sqrt{2}$

PART B – CHEMISTRY

31. $H_2O(s) \longrightarrow H_2O(\ell) \longrightarrow H_2O(\ell) \longrightarrow H_2O(g) \longrightarrow H_2O(g)$

1 kg at 273 K; 1 kg at 273 K; 1 kg at 373 K; at 373 K; at 383 K

$\Delta S = \Delta S_1 + \Delta S_2 + \Delta S_3 + \Delta S_4$

$$= \frac{334}{273} + 4.2\ell n\frac{373}{273} + \frac{2491}{373} + 2\ell n\frac{383}{373} = 9.267 \text{ kJ Kg}^{-1}\text{ K}^{-1}$$

32. $2C_{57}H_{110}O_6(s) + 163O_2(g) \longrightarrow 114CO_2(g) + 110H_2O(l)$

$$\frac{\text{Moles of } C_{57}H_{110}O_6}{2} = \frac{\text{Moles of } H_2O}{110}$$

$$\frac{\frac{445}{890}}{2} = \frac{\frac{\text{Mass of } H_2O}{18}}{110}$$

Mass of H_2O = 495 g

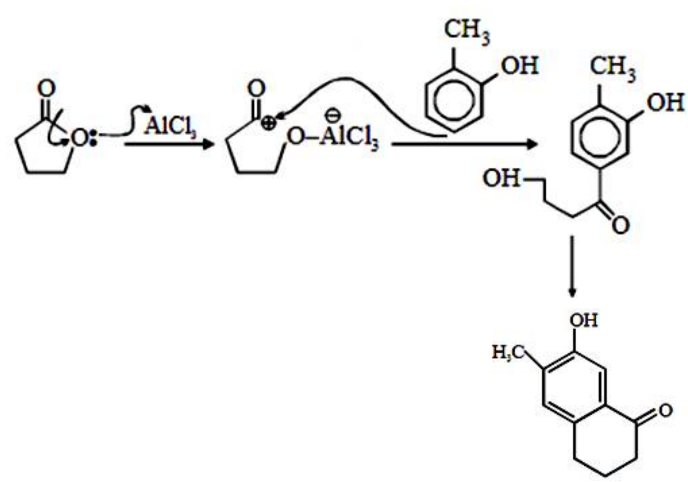
33. $Ph-\overset{O}{\overset{||}{C}}-CH_3 \xrightarrow{NaOH} \left[Ph-\overset{O}{\overset{||}{C}}-\overset{\ominus}{C}H_2 \longleftrightarrow Ph-\overset{O^-}{\overset{|}{C}}=CH_2\right]$

enolate ion

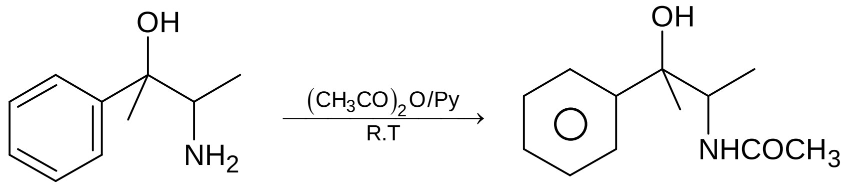
$Ph-\overset{O^\ominus}{\overset{|}{C}}=CH_2 + CH_3-\overset{O}{\overset{||}{C}}-H \longrightarrow CH_3-\overset{O^-}{\overset{|}{C}H}-CH_2-\overset{O}{\overset{||}{C}}-Ph \xrightarrow{H_2O} CH_3-\overset{OH}{\overset{|}{C}H}-CH_2-\overset{O}{\overset{||}{C}}-Ph$

34. Fact based

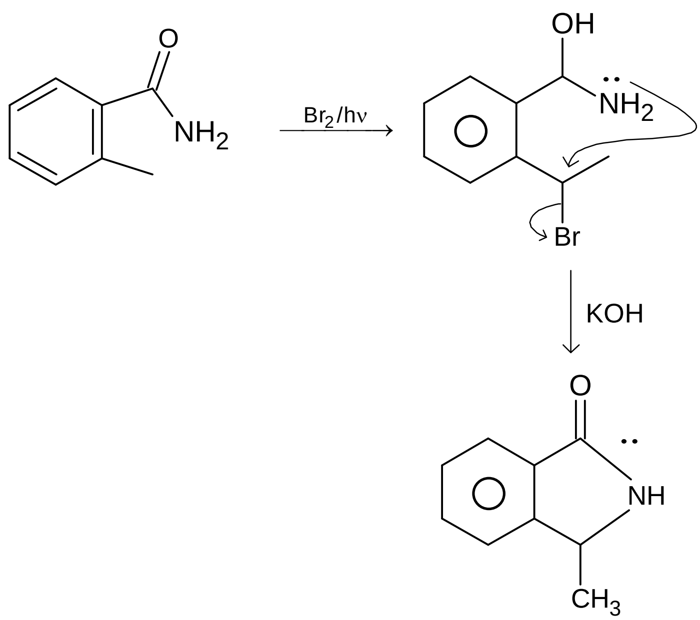
35.



36. Nucleophilicity of NH_2 > OH



37.



38. Acetonitrile is used as mobile phase for most of the reverse chromatography. Benzaldehyde is adsorbed on alumina.

39. The only alkali metal which forms nitride by reacting directly with N_2 is 'Li'.

40. As_2S_3 is a negatively charged sol. so $AlCl_3$ will be most effective.

41. As CN^- is a strong field ligand. $K_3[Co(CN)_6]$ will have maximum 'Δ'.

42. Fact based.

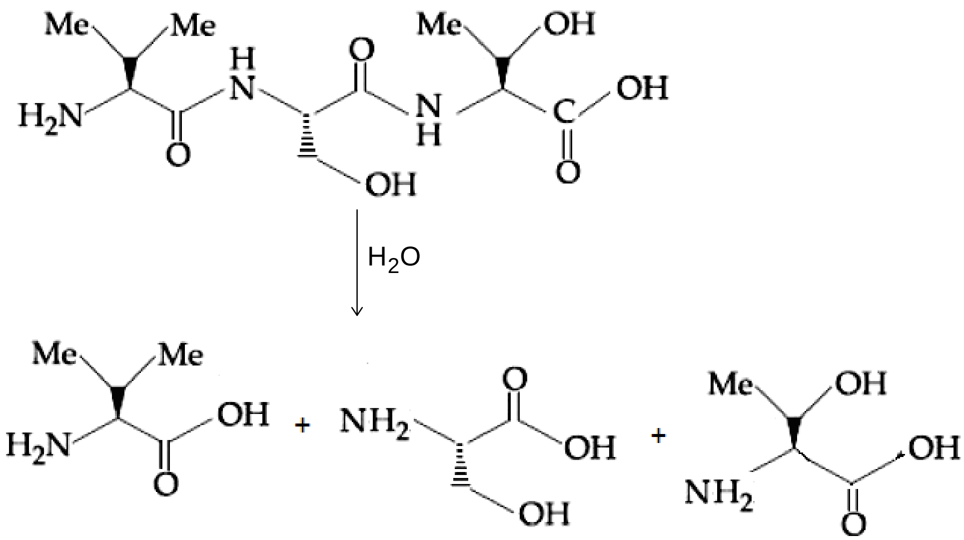
43. $A_2(g) + B_2(g) \underset{}{\overset{K_1}{\rightleftharpoons}} 2AB(g)$ ....(1)

$6AB(g) \overset{K_2}{\rightleftharpoons} 3A_2(g) + 3B_2(g)$ ....(2)

Reaction(2) = -3 × reaction(1)

$\therefore K_2 = \left(\frac{1}{K_1}\right)^3 \Rightarrow K_2 = K_1^{-3}$

44.



Val Ser Thr

45. $2A + B \longrightarrow$ products

Rate = $K[A]^x[B]^y$

$r = K[A]^x[B]^y$ - - - - (i)

$0.3 = K[A]^x[B]^y$ - - - - (1)

$2.4 = K[2A]^x[2B]^y$ - - - - (2)

$0.6 = K[2A]^x[B]^y$ - - - - (3)

From (1), (2) & (3)

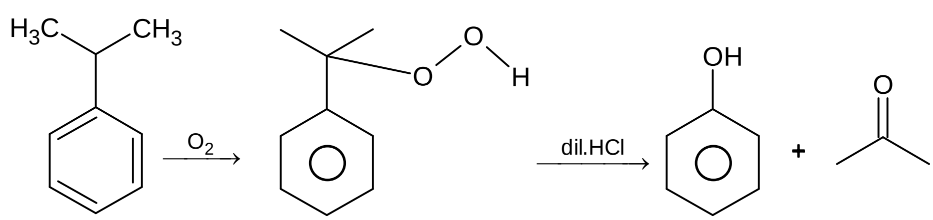
x = 1, y = 2

Overall order = 2 + 1 = 3

Order w.r.t A = 1

Order w.r.t B = 2

46.



47. ∵ $-COCH_3$ is present it will show both 2, 4-DNP & iodoform test.

Due to steric inhibition of resonance. I.P of 'N' is not involved in delocalization so coupling reaction will not take place.

48. $Zn(s) + Cu^{2+}(aq) \rightleftharpoons Zn^{2+}(aq) + Cu(s)$

$-nFE_{cell} = -RT\ell nK$

$\ell nK = \dfrac{2 \times 96500 \times 2}{8 \times 300} = 160.83$

$K = e^{160}$

49. Fact based.

50. $NO \longrightarrow NO^+$ $\qquad$ $N_2 \longrightarrow N_2^+$

B.O 0.5 3 $\qquad$ B.O 3 2.5

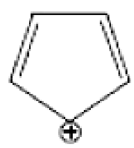
Para Dia $\qquad$ Dia Para

$O_2 \longrightarrow O_2^+$ $\qquad$ $O_2 \longrightarrow O_2^{2-}$

B.O 2 2.5 $\qquad$ B.O 2 1

Para Para $\qquad$ Para Dia

51. Refer Theory

52.  is anti aromatic

53. Refer theory

54. $4\,Al + 6\,ZnO \longrightarrow 2\,Al_2O_3 + 6\,Zn$

ΔH for the above reaction is –ve.

55. Due to weak metallic bonding.

56. Correct order of basic strength is

$NH_2(Et)_2 > EtNH_2 > NMC_3 > Ph - NH - CH_3$

57. 2^{nd} electron gain enthalpy of oxygen is positive.

58. $d = \dfrac{ZM}{N_a a^3}$

$= \dfrac{4 \times 63.55}{6.023 \times 10^{23} \times \left(x \times 10^{-8}\right)^3} = \dfrac{422}{x^3}\,gm/cm^3$

59. Let moles of H_2O separated as ice = x gm

$\Delta T_f = iK_f m$

$10 = 1 \times 1.86\ \dfrac{\frac{62}{62}}{\frac{250-x}{1000}}$

x = 64 gm

60. L_1 $\quad$ L_2 $\quad$ L_3

Green $\quad$ Blue $\quad$ Red absorbed wave length

Order of λ Red > Green > Blue

$L_3 > L_1 > L_2$

$\therefore$ Strength of ligand $\alpha\ \Delta\ \alpha\ 1/\lambda$

$\therefore$ Strength of ligand $L_2 > L_1 > L_3$

PART C – MATHEMATICS

61. $T_n = \dfrac{\left(3+(n-1)\times 3\right)\left(1^2+2^2+....+n^2\right)}{(2n+1)}$

$T_n = \dfrac{3.\dfrac{n^2(n+1)(2n+1)}{6}}{2n+1} = \dfrac{n^2(n+1)}{2}$

$S_{15} = \dfrac{1}{2}\sum_{n=1}^{15}\left(n^3+n^2\right) = \dfrac{1}{2}\left[\left(\dfrac{15(15+1)}{2}\right)^2 + \dfrac{15\times 16\times 31}{6}\right]$

$= 7820$

62. $\lim\limits_{x\to 0^+} \dfrac{x([x]+|x|)\sin[x]}{|x|}$

$x \to 0^-$

$\left.\begin{matrix}[x]=-1\\ |x|=-x\end{matrix}\right\} \Rightarrow \lim\limits_{x\to 0^-}\dfrac{x(-x-1)\sin(-1)}{-x} = -\sin 1$

63. $f(xy)=f(x).f(y)$

$f(0)=1$ as $f(0)\neq 0$

$\Rightarrow f(x)=1$

$\dfrac{dy}{dx}=f(x)=1$

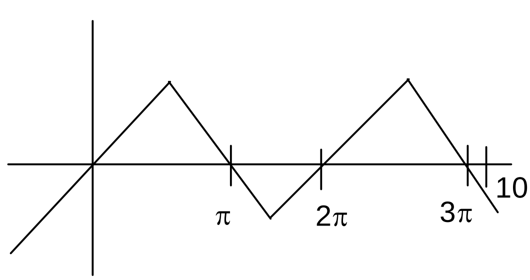
$\Rightarrow y=x+c$

At, $x=0,\ y=1 \Rightarrow c=1$

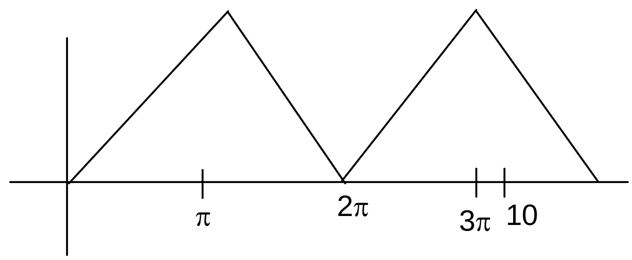
$y=x+1$

$\Rightarrow y\left(\dfrac{1}{4}\right)+y\left(\dfrac{3}{4}\right)=\dfrac{1}{4}+1+\dfrac{3}{4}+1=3$

64.



$x=\sin^{-1}(\sin 10)=3\pi-10$



$y = \cos^{-1}(\cos 10) = 4\pi - 10$
$y - x = \pi$

65. $\sin x - \sin 2x + \sin 3x = 0$

$\Rightarrow (\sin x + \sin 3x) - \sin 2x = 0$

$\Rightarrow 2\sin x . \cos x - \sin 2x = 0$

$\Rightarrow \sin 2x(2\cos x - 1) = 0$

$\Rightarrow \sin 2x = 0$ or $\cos x = \frac{1}{2}$ $\Rightarrow x = 0, \frac{\pi}{3}$

66. $z_0 = \omega$ or ω^2 (where ω is a non – real cube root of unity)

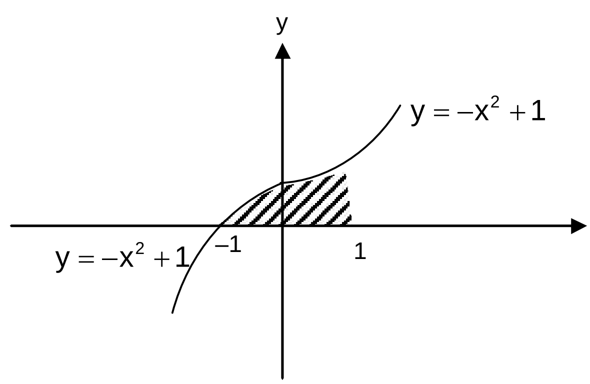
$z = 3 + 6i(\omega)^{81} - 3i(\omega)^{93}$

$z = 3 + 3i$

$\Rightarrow \arg z = \frac{\pi}{4}$

67. The graph is a follows

$\int_{-1}^{0}(-x^2 + 1)dx + \int_{0}^{1}(x^2 + 1)dx = 2$



68. $P_1 = x - 4y + 7z - g = 0$

$P_2 = 3x - 5y - h = 0$

$P_3 = -2x + 5y - 9z - k = 0$

Here $\Delta = 0$

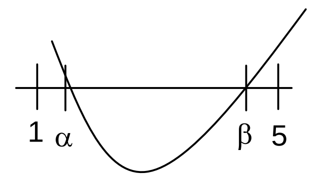
$2P_1 + P_2 + P_3 = 0$ when $2g + h + k = 0$

69. $(1 - t^6)^3 (1 - t)^{-3}$

$(1 - t^{18} - 3t^6 + 3t^{12})(1 - t)^{-3}$

$\Rightarrow$ coefficient of t^4 in $(1 - t)^{-3}$ is ${}^{3+4-1}C_4 = {}^6C_2 = 15$

70. $x^2 - mx + 4 = 0$
$\alpha, \beta \in [1, 5]$



(1) $D > 0 \Rightarrow m^2 - 16 > 0$
$\Rightarrow m \in (-\infty, -4) \cup (4, \infty)$

(2) $f(1) \geq 0 \Rightarrow 5 - m \geq 0 \Rightarrow m \in (-\infty, 5]$

(3) $f(5) \geq 0 \Rightarrow 29 - 5m \geq 0 \Rightarrow m \in \left(-\infty, \frac{29}{5}\right]$

(4) $1 < \frac{-b}{2a} < 5 \Rightarrow 1 < \frac{m}{2} < 5 \Rightarrow m \in (2, 10)$
$\Rightarrow m \in (4, 5)$
No option correct : Bonus

* If we consider $\alpha, \beta \in (1, 5)$ then option (1) is correct.

71. Let $A(\alpha, 0)$ and $B(0, \beta)$ be the vectors of the given triangle AOB
$\Rightarrow |\alpha\beta| = 100$
$\Rightarrow$ Number of triangles
$= 4 \times$ (number of divisors of 100)
$= 4 \times 9 = 36$

72. $a = A + 6d$
$b = A + 10d$
$c = A + 12d$
a, b, c are in G.P.
$\Rightarrow (A + 10d)^2 = (A + 6d)(a + 12d)$
$\Rightarrow \frac{A}{d} = -14$

$$\frac{a}{c} = \frac{A + 6d}{A + 12d} = \frac{6 + \frac{A}{d}}{12 + \frac{A}{d}} = \frac{6 - 14}{12 - 14} = 4$$

73. $[\sim(\sim p \vee q) \wedge (p \wedge r)] \cap (\sim q \wedge r)$
$\equiv [(p \wedge \sim q) \vee (p \wedge r)] \wedge (\sim q \wedge r)$
$\equiv [p \wedge (\sim q \vee r)] \wedge (\sim q \wedge r)$
$\equiv p \wedge (\sim q \wedge r)$
$\equiv (p \wedge r) \sim q$

74. Vector along the normal to the plane containing the lines $\frac{x}{3}=\frac{y}{4}=\frac{z}{2}$ and $\frac{x}{4}=\frac{y}{2}=\frac{z}{3}$ is $\left(8\hat{i}-\hat{j}-10\hat{k}\right)$.

Vector perpendicular to the vectors $2\hat{i}+3\hat{j}+4\hat{k}$ and $8\hat{i}-\hat{j}-10\hat{k}$ is $26\hat{i}-52\hat{j}+26\hat{k}$

So, required plane is $26x-52y+26z=0 \Rightarrow x-2y+z=0$

75. $\sum\left(x_i+1\right)^2=9n$ ....(1)

$\sum\left(x_i-1\right)^2=5n$ ....(2)

$(1)+(2)\Rightarrow\sum\left(x_i^2+1\right)=7n$

$\Rightarrow\frac{\sum x_i^2}{n}=6$

(1) . (2) $\Rightarrow 4\sum x_i=4n$

$\Rightarrow\sum x_i=n$

$\Rightarrow\frac{\sum x_i}{n}=1$

$\Rightarrow$ variance = 6 – 1 = 5

$\Rightarrow$ standard diviation $=\sqrt{5}$

76. $|A|=e^{-t}\begin{vmatrix}1 & \cos t & \sin t\\ 1 & -\cos t-\sin t & -\sin t+\cos t\\ 1 & 2\sin t & -2\cos t\end{vmatrix}$

$=e^{-t}\left[5\cos^2 t+5\sin^2 t\right]\forall t\in R$

$=5e^{-t}\neq 0\,\forall t\in R$

77. $\int\frac{5x^8+7x^6}{\left(x^2+1+2x^7\right)^2}dx$

$=\int\frac{5x^{-6}+7x^{-8}}{\left(\frac{1}{x^7}+\frac{1}{x^5}+2\right)^2}dx=\frac{1}{2+\frac{1}{x^5}+\frac{1}{x^7}}+C$

As $f(0)=0,\ f(x)=\frac{x^7}{2x^7+x^2+1}$

$f(1)=\frac{1}{4}$

78. $\left|f(x)-f(y)\right|\le 2|x-y|^{3/2}$

divide both side by $|x-y|$

$\left|\frac{f(x)-f(y)}{x-y}\right|\le 2.|x-y|^{1/2}$

Apply limit $x \to y$

$|f'(y)| \le 0 \Rightarrow f'(y) = 0 \Rightarrow f(y) = c \Rightarrow f(x) = 1$

$\int_0^1 1.\,dx = 1$

79. $\frac{dx}{dt} = 3\sec^2 t$

$\frac{dy}{dt} = 3\sec t \tan t$

$\frac{dy}{dx} = \frac{\tan t}{\sec t} = \sin t$

$\frac{d^2y}{dx^2} = \cos t \frac{dt}{dx}$

$= \frac{\cos t}{3\sec^2 t} = \frac{\cos^3 t}{3} = \frac{1}{3.2\sqrt{2}} = \frac{1}{6\sqrt{2}}$

80. | a_1 | a_2 | a_3 |
|---|---|---|

Number of numbers $= 5^3 - 1$

a_4	a_1	a_2	a_3

2 ways for a_4

Numbers of numbers $= 2 \times 5^3$

Required number0020 $= 5^3 + 2 \times 5^3 - 1$

= 374

81. $x^2 + y^2 - 16x - 20y + 164 = r^2$

$A(8,10), R_1 = r$

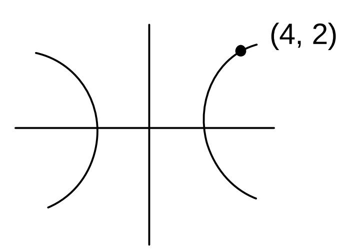
$(x-4)^2 + (y-7)^2 = 36$

B (4, 7), $R_2 = 6$

$|R_1 - R_2| < AB < R_1 + R_2$

$\Rightarrow 1 < r < 11$

82. Given hyperbola is



$\frac{x^2}{4} - \frac{y^2}{b^2} = 1$

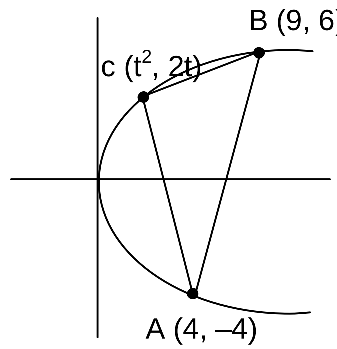
Satisfying the point (4, 2)

$\Rightarrow b^2 = \frac{4}{3}$

$\Rightarrow e = \frac{2}{\sqrt{3}}$

83. For maximum area, tangent at the point c must be parallel to chord BC.

$\therefore \ t = \frac{1}{2}$



84. Equation of AB is $3x - 2y + 6 = 0$

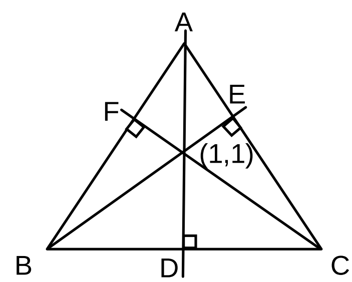
Equation of AC is $4x + 5y - 20 = 0$.

Equation of BE is $2x + 3y - 5 = 0$

Equation of CF is $5x - 4y - 1 = 0$

$\Rightarrow$ Equation of BC is

$26x - 122y = 1675$



85. E_1 : Event of drawing a Red ball and placing a green ball in the bag

E_2 : Event of drawing a green ball and placing a red ball in the bag

E : Event of drawing a red ball in second draw $P(E) = P(E_1) \times P\left(\frac{E}{E_1}\right) + P(E_2) \times P\left(\frac{E}{E_2}\right)$

$= \frac{5}{7} \times \frac{4}{7} + \frac{2}{7} \times \frac{6}{7} = \frac{32}{49}$

86. Line $x = ay + b, z = cy + d$

$\Rightarrow \frac{x-b}{a} = \frac{y}{1} = \frac{z-d}{c}$

Line $x = a'z + b', y = c'z + d'$

$\Rightarrow \frac{x-b'}{a'} = \frac{y-d'}{c'} = \frac{z}{1}$

Given both the lines are perpendicular

$\Rightarrow aa' + c' + c = 0$

87. Projection of $\vec{b}$ on $\vec{a} = \frac{\vec{a}.\vec{b}}{|\vec{a}|} = |\vec{a}|$

$\Rightarrow b_1 + b_2 = 2$ …………(1)

and $(\vec{a} + \vec{b}) \perp \vec{c} \Rightarrow (\vec{a} + \vec{b}).\vec{c} = 0$

$\Rightarrow 5b_1 + b_2 = -10$ ………..(2)

from (1) and (2) $\Rightarrow b_1 = -3$ and $b_2 = 5$

then $|\vec{b}| = \sqrt{b_1^2 + b_2^2 + 2} = 6$

88. D must be perfect square

$\Rightarrow 121 - 24\alpha = \lambda^2$

$\Rightarrow$ maximum value of α is 5

$\alpha = 1 \Rightarrow \lambda \notin I$

$\alpha = 2 \Rightarrow \lambda \notin I$

$\alpha = 3 \Rightarrow \lambda \in I$ $\quad\quad \Rightarrow$ 3 integral values

$\alpha = 4 \Rightarrow \lambda \in I$

$\alpha = 5 \Rightarrow \lambda \in I$

89. $f(x) = 2\left(1 + \frac{1}{x-1}\right)$

$f'(x) = -\frac{2}{(x-1)^2}$

$\Rightarrow$ f is one – one but not onto

90. $\frac{1}{\sqrt{2k}} \int_0^{\pi/3} \frac{\tan\theta}{\sqrt{\sec\theta}} d\theta = \frac{1}{\sqrt{2k}} \int_0^{\pi/3} \frac{\sin\theta}{\sqrt{\cos\theta}} d\theta$

$= -\frac{1}{\sqrt{2k}} 2\sqrt{\cos\theta}\Big|_0^{\pi/3} = -\frac{\sqrt{2}}{\sqrt{k}}\left(\frac{1}{\sqrt{2}} - 1\right)$

$\Rightarrow k = 2$