PART –A (PHYSICS)

1. A particle moves from the point $\left(2.0\hat{i}+4.0\hat{j}\right)$m, at t = 0 with an initial velocity $\left(5.0\hat{i}+4.0\hat{j}\right)$ $ms^{-1+.}$ It is acted upon by a constant force which produces a constant acceleration $\left(4.0\hat{i}+4.0\hat{j}\right)ms^{-2}$. What is the distance of the particle from the origin at time 2s?

(A) 15m
(B) $20\sqrt{2}$ m
(C) 5m
(D) $10\sqrt{2}$ m

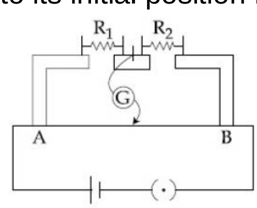
2. A thermometer graduated according to a linear scale reads a value x_0 when in contact with boiling water, and $x_0/3$ when in contact with ice. What is the temperature of an object in ^{0}C, if this thermometer in the contact with the object reads $x_0/2$?

(A) 25
(B) 60
(C) 40
(D) 45

3. A galvanometer having a resistance of 20 Ω and 30 divisions on both sides has figure of merit 0.005 ampere /division. The resistance that should be connected in series such that it can be used as a voltmeter upto 15 volt is:

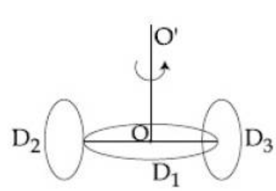
(A) 100 Ω
(B) 120 Ω
(C) 80 Ω
(D) 125 Ω

4. In the experimental setup of metre bridge shown in the figure, the null point is obtained at a distance of 40cm from A. If a 10Ω resistor is connected in series with R_1, the null point shifts by 10cm. The resistance that should be connected in parallel with $(R_1 + 10)\Omega$ such that the null point shifts back to its initial position is:



(A) 20Ω
(B) 40Ω
(C) 60Ω
(D) 30Ω

5. A circular disc D_1 of mass M and radius R has two identical discs D_2 and D_3 of the same mass M and radius R attached rigidly as its opposite ends (see figure). The moment of inertia of the system about the axis OO', passing through the centre of D_1 as shown in the figure, will :



(A) MR^2
(B) $3MR^2$
(C) $\frac{4}{5}MR^2$
(D) $\frac{2}{3}MR^2$

6. The magnitude of torque on a particle of mass 1kg is 2.5 Nm about the origin. If the force acting on it is IN, and the distance of the particle from the origin is 5m, the angle between the force and the position vector is (in radians):

(A) $\frac{\pi}{6}$ (B) $\frac{\pi}{3}$

(C) $\frac{\pi}{8}$ (D) $\frac{\pi}{4}$

7. A copper wire is wound on a wooden frame, whose shape is that of an equilateral triangle. If the linear dimension of each side of the frame is increased by a factor of 3, keeping the number of turns of the coil per unit length of the frame the same, then self inductance of the coil:

(A) decreases by a factor of 9 (B) increases by a factor of 27

(C) increases by a factor of 3 (D) decreases by a factor of $9\sqrt{3}$

8. A particle of mass m is moving in a straight line with momentum p. Starting at time t = 0, a force F = kt acts in the same direction on the moving particle during time interval T so that its momentum changes from p to 3p. Here k is a constant. The value of T is:

(A) $2\sqrt{\frac{k}{p}}$ (B) $2\sqrt{\frac{p}{k}}$

(C) $\sqrt{\frac{2k}{p}}$ (D) $\sqrt{\frac{2p}{k}}$

9. A paramagnetic substance in the form of a cube with sides 11 cm has a magnetic dipole moment of 20 x 10^{-6} J/T when a magnetic intensity of 60 x 10^{3} A/m is applied. Its magnetic susceptibility is:

(A) 3.3 x 10^{-2} (B) 40.3 x 10^{-2}

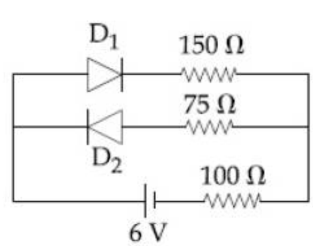
(C) 2.3 x 10^{-2} (D) 3.3 x 10^{-4}

10. A simple pendulum of length 1 m is oscillating with an angular frequency 10 rad/s. The support of the pendulum starts oscillating up and down with a small angular frequency of 1 rad/s and an amplitude of 10^{-2} m. The relative change in the angular frequency of the pendulum is best given by:

(A) 10^{-3} rad/s (B) 1 rad/s

(C) 10^{-1} rad/s (D) 10^{-5} rad/s

11. The circuit shown below contains two ideal diodes, each with a forward resistance of 50Ω. If the battery voltage is 6V, the current through the 100Ω resistance (in Amperes) is:



(A) 0.036 (B) 0.020

(C) 0.027 (D) 0.030

12. An electric field of 1000V/m is applied to an electric dipole at angle of 45°. The value of electric dipole moment is 10^{-29} C.m. What is the potential energy of the electric dipole?

(A) $-20\times10^{18}\,J$
(B) $-7\times10^{-27}\,J$
(C) $-10\times10^{-29}\,J$
(D) $-9\times10^{-20}\,J$

13. A metal bal of mass 0.1 kg is heated upto 500^0C and dropped into a vessel of heat capacity 800 JK^{-1} and containing 0.5 kg water. The initial temperature of water and vessel is 30^0C. What is the approximate percentage increment in the temperature of the water? [Specific heat Capacities of water and metal are, respectively $4200\,Jkg^{-1}K^{-1}$ and 400 $jkg^{-1}K^{-1}$]

(A) 15%
(B) 30%
(C) 25%
(D) 20%

14. The region between y = 0 and y = d contains a magnetic field $\vec{B}=B\hat{z}$. A particle of mass m and charge q enters the region with a velocity $\vec{v}=v\hat{i}$. If d = $\frac{mv}{2qB}$, the acceleration of the charged particle at the point of its emergence at the other side is:

(A) $\frac{qvB}{m}\left(\frac{1}{2}\hat{i}-\frac{\sqrt{3}}{2}\hat{j}\right)$
(B) $\frac{qvB}{m}\left(\frac{\sqrt{3}}{2}\hat{i}+\frac{1}{2}\hat{j}\right)$
(C) $\frac{qvB}{m}\left(\frac{-\hat{j}+\hat{i}}{\sqrt{2}}\right)$
(D) $\frac{qvB}{m}\left(\frac{\hat{j}+\hat{i}}{\sqrt{2}}\right)$

15. A pendulum is executing simple harmonic motion and its maximum kinetic energy is K_1. If the length of the pendulum is doubled and it performs simple harmonic motion with the same amplitude as in the first case, its maximum kinetic energy is K_2 then:

(A) $K_2=2K_1$
(B) $K_2=\frac{K_1}{2}$
(C) $K_2=\frac{K_1}{4}$
(D) $K_2=K_1$

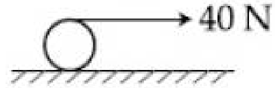
16. Two rods A and B of identical dimensions are at temperature 30°C. If a heated upto 180°C and B upto T^0C, then the new lengths are the same. If the ratio of the coefficients of linear expansion of A and B is 4:3, then the value of T is:

(A) 230^0C
(B) 270°C
(C) 200°C
(D) 250°C

17. If speed (V), acceleration (A) and force (F) are considered as fundamental units, the dimension of Young's modulus will be:

(A) $V^{-2}A^2F^{-2}$
(B) $V^{-2}A^2F^2$
(C) $V^{-4}A^{-2}F$
(D) $V^{-4}A^2F$

18. A string is wound around a hollow cylinder of mass 5 kg and radius 0.5m. If the string is now pulled with a horizontal force of 40 N, and the cylinder is rolling without slipping on a horizontal surface (see figure), then the angular acceleration of the cylinder will be (Neglect the mass and thickness of the string)

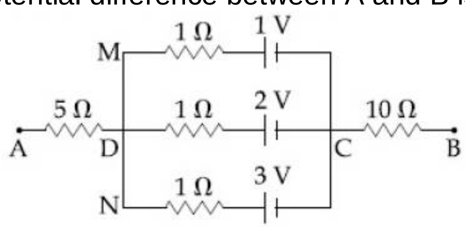


(A) 20 rad/s^2
(B) 16 rad/s^2
(C) 12 rad/s^2
(D) 10 rad/s^2

19. A 27 mW lager beam has a cross – sectional area of 10 mm^2. The magnitude of the maximum electric field in this electromagnetic wave is given by:
[Given permittivity of space $\epsilon_0 = 9 \times 10^{-12}$ SI units, speed of light c = 3 x 10^8 m/s]

(A) 2 kV/m
(B) 0.7 kV/m
(C) 1 kV/m
(D) 1.4 kV/m

20. In the circuit shown, the potential difference between A and B is:

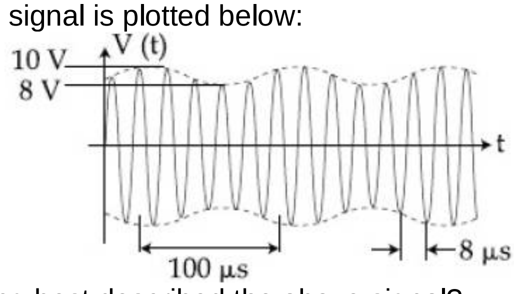


(A) 1V
(B) 2V
(C) 3V
(D) 6V

21. The mass and the diameter of a planet are three times the respective values for the Earth. The period of oscillation of a simple pendulum on the Earth is 2s. The period of oscillation of the same pendulum on the planet would be:

(A) $\frac{\sqrt{3}}{2}$s
(B) $\frac{2}{\sqrt{3}}$s
(C) $\frac{3}{2}$s
(D) $2\sqrt{3}$ s

22. An amplitude modulated signal is plotted below:



Which one of the following best described the above signal?

(A) $\left(9+\sin\left(2.5\pi\times10^5 t\right)\right)\sin\left(2\pi+10^4 t\right)$V
(B) $\left(1+9\sin\left(2\pi\times10^4 t\right)\right)\sin\left(2.5\pi\times10^5 t\right)$V
(C) $\left(9+\sin\left(2\pi\times10^4 t\right)\right)\sin\left(2.5\pi\times10^5 t\right)$V
(D) $\left(9+\sin\left(4\pi\times10^4 t\right)\right)\sin\left(5\pi\times10^5 t\right)$V

23. In a process, temperature and volume of one mole of an ideal monoatomic gas are varied according to the relation VT = K, where I is a constant. In this process the temperature of the gas is increased by ΔT. The amount of heat absorbed by gas is (R is gas constant)

(A) $\frac{1}{2}R\Delta T$
(B) $\frac{1}{2}KR\Delta T$
(C) $\frac{3}{2}R\Delta T$
(D) $\frac{2K}{3}\Delta T$

24. When 100g of a liquid A at 100^0C is added to 50g of a liquid B at temperature 75^oC, the temperature of the mixture becomes 90^oC. The temperature of the mixture, if 100g of liquid A at 100^oC is added to 50g of liquid B at 50^oC, will be:

(A) 85^oC
(B) 60^oC
(C) 80^oC
(D) 70^oC

25. In a hydrogen like atom, when an electron jumps from the M – shell to the L-shell the wavelength of emitted radiation is λ. If an electron jumps from N-shell to the L-shell the wavelength of emitted radiation will be:

(A) $\frac{27}{20}\lambda$
(B) $\frac{16}{25}\lambda$
(C) $\frac{25}{16}\lambda$
(D) $\frac{20}{27}\lambda$

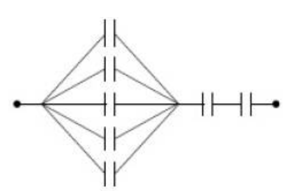
26. A monochromatic light is incident at a certain angle on an equilateral triangular prism and suffers minimum deviation. If the refractive index of the material of the prism is $\sqrt{3}$, then the angle of incidence

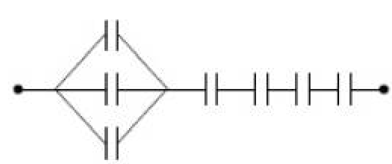
(A) 90^o
(B) 30^o
(C) 60^o
(D) 45^o

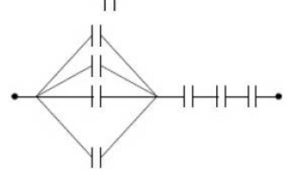
27. In a double – slit experiment, green light (5303 $\overset{0}{A}$) falls on a double slit having a separation of 19.44 μm and a width of 4.05μm. The number of bright fringes between the first and the second diffraction minima is:

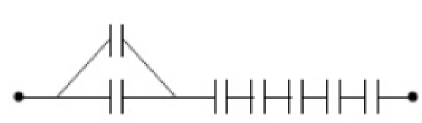
(A) 10
(B) 05
(C) 04
(D) 09

28. Seven capacitors, each of the capacitance 2μF, are to be connected in a configuration to obtain an effective capacitance of $\left(\frac{6}{13}\right)$ μF. Which of the combinations, shown in figures below, will achieve the desired value?

(A)


(B)


(C)


(D)


29. A particle of mass m and charge q is in an electric and magnetic field given by:

$$\vec{E} = 2\hat{i} + 3\hat{j}; B = 4\hat{j} + 6\hat{k}$$

The charged particle is shifted from the origin to the point P(x = 1; y = 1) along a straight path. The magnitude of the total work done is:

(A) (0.35)q
(B) 5q
(C) (2.5)q
(D) (0.15)q

30. In a photoelectric experiment, the wavelength of the light incident on a metal is changed from 300 nm to 400 nm. The decrease in the stopping potential is close to $\left(\frac{hc}{e} = 1240\ \text{nm} - \text{V}\right)$

(A) 0.5 V
(B) 1.5 V
(C) 1.0 V
(D) 2.0 V

PART –B (CHEMISTRY)

31. The reaction:
$MgO(s) + C(s) \rightarrow Mg(s) + CO(g)$, for which $\Delta_r H^0 = +491.1$ kJ mol^{-1} and $\Delta_r S^0 = 198.0$ JK^{-1} mol^{-1} is not feasible at 298 K. Temperature above which reaction will be feasible is:

(A) 2040.5 K
(B) 1890.0K
(C) 2480. K
(D) 2380.K

32. The correct match between Item I and Item II is

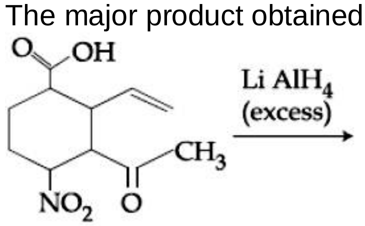
Item I	Item II
A. Allosteric effect	P. Molecule binding to the active site of enzyme
B. Competitive inhibitor	Q. Molecule crucial for communication in the body
C. Receptor	R. Molecule binding to a site other than the active site of enzyme
D. Poison	S. Molecule binding to the enzyme covalently

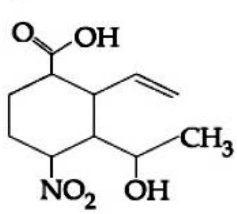
(A) $A \rightarrow R$, $B \rightarrow P$, $C \rightarrow Q$, $D \rightarrow S$
(B) $A \rightarrow P$, $B \rightarrow R$, $C \rightarrow Q$, $D \rightarrow S$
(C) $A \rightarrow R$, $B \rightarrow P$, $C \rightarrow S$, $D \rightarrow Q$
(D) $A \rightarrow P$, $B \rightarrow R$, $C \rightarrow S$, $D \rightarrow Q$

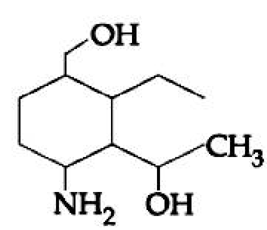
33. The coordination number of Th in $K_4\left[Th(C_2O_4)_4(OH_2)_2\right]$ is: $\left(C_2O_4^{2-} = \text{Oxalato}\right)$

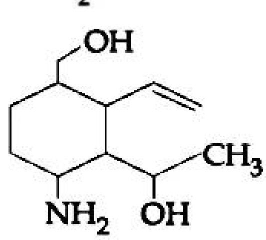
(A) 14
(B) 6
(C) 8
(D) 10

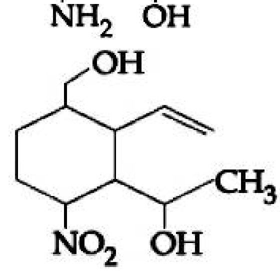
34. The major product obtained in the following reaction is:



(A)


(B)


(C)


(D)


35. The standard reaction Gibbs energy for a chemical reaction at an absolute temperature T is given by

$\Delta_r G^0 = A - BT$

Where A and B are non – zero constant. Which of the following is TRUE about this reaction?

(A) Endothermic if A > 0
(B) Exothermic if A > 0 and B < 0
(C) Endothermic if A < 0 and B > 0
(D) Exothermic if B < 0

36. The radius of the largest sphere which fits properly at the centre of the edge of a body centered cubic unit cell is: (Edge length is represented by 'a')

(A) 0.0027a
(B) 0.047 a
(C) 00137a
(D) 0.07a

37. The hydride that is NOT electron deficient is:

(A) SiH_4
(B) B_2H_6
(C) GaH_3
(D) AlH_3

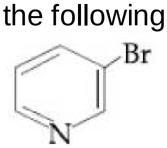
38. Given the equilibrium constant:
K_C of the reaction:
$Cu(s) + 2Ag + (aq) \rightarrow Cu^2 + (aq) + 2Ag(s)$ is 10 x $10^{15=}$, calculate the E^{θ}_{cell} of the reaction of 298 K $\left[2.303\frac{RT}{F} \text{ at } 298K = 0.059V\right]$

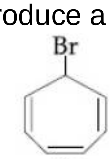
(A) 0.04736 mV
(B) 0.4736 mV
(C) 0.4736 V
(D) 0.04736 V

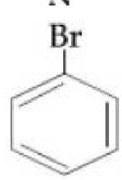
39. The correct option with respect to the Pauling electronegativity values of the elements is:

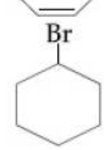
(A) Te > Xe
(B) Ga > Ge
(C) Si > Al
(D) P > S

40. Which of the following compounds will produce a precipitate with $AgNO_3$?

(A)


(B)


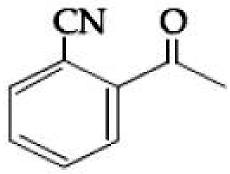
(C)


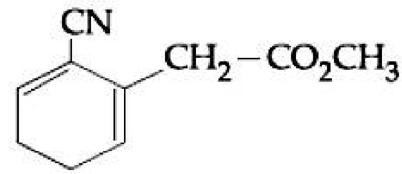
(D)


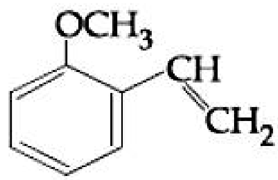
41. The de Broglie wavelength (λ) associated with a photoelectron varies with the frequency (v) of the incident radiation as, [v_0 is threshold frequency]:

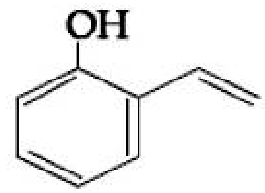
(A) $\lambda \propto \frac{1}{(v - v_0)}$
(B) $\lambda \propto \frac{1}{(v - v_0)^{\frac{1}{4}}}$
(C) $\lambda \propto \frac{1}{(v - v_0)^{\frac{3}{2}}}$
(D) $\lambda \propto \frac{1}{(v - v_0)^{\frac{1}{2}}}$

42. Which of the following compounds reacts with ethylmagnesium bromide and also decolourizes bromine water solution:

(A) 

(B) 

(C) 

(D) 

43. In the following compound

The favourable site/s for protonation is/are

(A) a and e
(B) b, c and d
(C) a and d
(D) a

44. Taj Mahal is being slowly disfigured and discoloured. This is primarily due to:

(A) global warming
(B) acid rain
(C) water pollution
(D) soil pollution

45. The relative stability of +1 oxidation state of group 13 elements follows the order:

(A) $Al < Ga < Tl < In$
(B) $Tl < In < Ga < Al$
(C) $Ga < Al < In < Tl$
(D) $Al < Ga < In < Tl$

46. For the equilibrium

$2H_2O \rightleftharpoons H_3O^+ + OH^-$, the value of ΔG^0 at 298 K is approximately:

(A) 100 kJ mol^{-1}
(B) – 80 kJ mol^{-1}
(C) 80 kJ mol^{-1}
(D) –100 kJ mol^{-1}

47. The reaction that does NOT define calcinations is:

(A) $Fe_2O_3.XH_2O \xrightarrow{\Delta} Fe_2O_3 + XH_2O$

(B) $2Cu_2S + 3O_2 \longrightarrow 2Cu_2O + 2SO_2$

(C) $ZnCO_3 \xrightarrow{\Delta} ZnO + CO_2$

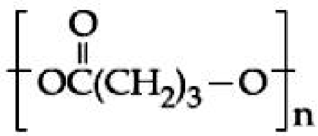
(D) $CaCO_3.MgCO_3 \xrightarrow{\Delta} CaO + MgO + 2CO_2$

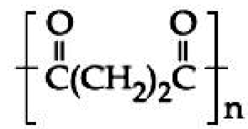
48. A compound 'X' on treatment with Br_2/NaOH, provided C_3H_9N, which gives positive carbylamine test. Compound 'X' is:

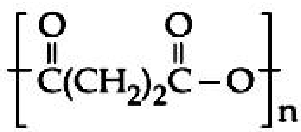
(A) $CH_3COCH_2NHCH_3$
(B) $CH_3CH_2COCH_2NH_2$
(C) $CH_3CH_2CH_2CONH_2$
(D) $CH_3CON(CH_3)_2$

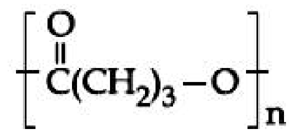
49. Among the colloids cheese (C), milk (M) and smoke (S), the correct combination of the dispersed phase and dispersion medium, respectively is:
(A) C: liquid in solid; M: liquid in solid; S: solid in gas
(B) C : liquid in solid; M: liquid in liquid; S: solid in gas
(C) C : solid in liquid ; M : liquid in liquid ; S : gas in solid
(D) C : solid in liquid ; M : solid in liquid; S : solid in gas

50. The homopolymer formed from 4 – hydroxybutanoic acid is:

(A) 

(B) 

(C) 

(D) 

51. K_2HgI_4 is 40% ionized in aqueous solution. The value of its van't Hoff factor (i) is:
(A) 1.6
(B) 1.8
(C) 2.0
(D) 2.2

52. 25 mL of the given HCl solution requires 20 mL of 0.1 M sodium carbonate solution. What is the volume of this HCl solution required to titrate 30 mL of 0.0 M aqueous $NaOH$ solution?
(A) 25 mL
(B) 75 mL
(C) 50 mL
(D) 12.5 mL

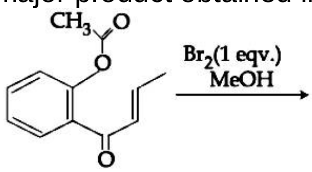
53. The reaction $2X \rightarrow B$ is a zeroth order reaction. If the initial concentration of X is 0.2M, the half life is 6 h. When the initial concentration of X is 0.5 M, the time required to reach its final concentration of 0.2 M will be:
(A) 9.0 h
(B) 12.0 h
(C) 18.0 h
(D) 7.2 h

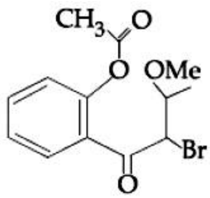
54. Match the following items in column I with the corresponding items in column II.

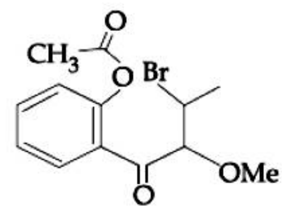
Column I	**Column II**
I. $Na_2CO_3 . 10 H_2O$	A. Portland cement ingredient
II. $Mg(HCO_3)_2$	B. Castner – Kellner process
III. $NaOH$	C. Solvay process
IV. $Ca_3Al_2O_6$	D. Temporary hardness

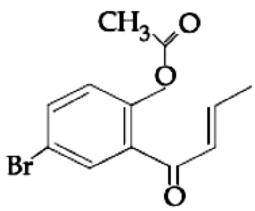
(A) I – B, II – C, III – A, IV – D
(B) I – C, II – B, III – D, IV – A
(C) I – D, II – A, III – B, IV – C
(D) I – C, II – D, III – B, IV – A

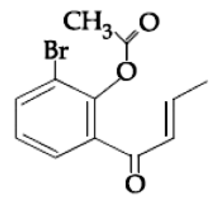
55. The major product obtained in the following conversion is:



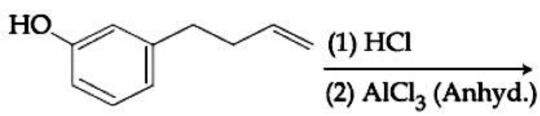
(A) 

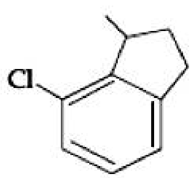
(B) 

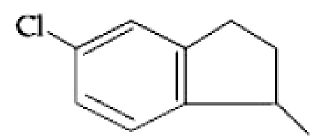
(C) 

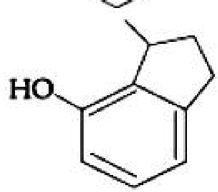
(D) 

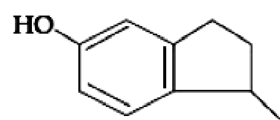
56. The major product of the following reaction is:



(A) 

(B) 

(C) 

(D) 

57. The higher concentration of which gas in air can cause stiffness of flower buds?

(A) NO_2
(B) CO_2
(C) SO_2
(D) CO

58. The correct match between Item I and Item II is

Item I	Item II
A. Ester test	P. Tyr
B. Carbylamine test	Q. Asp
C. Phthalein dye test	R. Ser
	S. Lys

(A) A – Q, B – S, C – P
(B) A – R, B – Q, C – P
(C) A – R, B – S, C – Q
(D) A – Q, B – S, C – R

59. The number of bridging CO ligand(s) and Co-Co bond(s) in $Co_2(CO)_8$ respectively are:

(A) 2 and 1
(B) 2 and 0
(C) 0 and 2
(D) 4 and 0

60. $$\underline{A} \xrightarrow{4KOH, O_2} \underset{(Green)}{2\underline{B}} + 2H_2O$$

$$3\underline{B} \xrightarrow{4HCl} \underset{(Purple)}{2\underline{C}} + MnO_2 + 2H_2O$$

$$2\underline{C} \xrightarrow{H_2O, KI} 2\underline{A} + 2KOH + \underline{D}$$

In the above sequence of reactions, A and D, respectively are:

(A) Ki and $KMnO_4$
(B) MnO_2 and KIO_3
(C) KIO_3 and MnO_2
(D) KI and K_2MnO_4

PART–C (MATHEMATICS)

61. $\lim_{x\to 0}\frac{x\cot(4x)}{\sin^2 x\cot^2(2x)}$ is equal to:

(A) 0
(B) 2
(C) 4
(D) 1

62. All x satisfying the inequality $\left(\cot^{-1}x\right)^2-7\left(\cot^{-1}x\right)+10>0$, lie in the interval:

(A) $(-\infty,\cot 5)\cup(\cot 4,\cot 2)$
(B) $(\cot 2,\infty)$
(C) $(-\infty,\cot 5)\cup(\cot 2,\infty)$
(D) $(\cot 5,\cot 4)$

63. If a hyperbola has length of its conjugate axis equal to 5 and the distance between its foci is 13, then the eccentricity of the hyperbola is:

(A) $\frac{13}{12}$
(B) 2
(C) $\frac{13}{6}$
(D) $\frac{13}{8}$

64. If the area of the triangle whose one vertex is at the vertex of the parabola, $y^2+4\left(x-a^2\right)=0$ and the other two vertices are the points of intersection of the parabola and y – axis, is 250 sq. units, then a value of ‘a’ is:

(A) $5\sqrt{5}$
(B) $5\left(2^{1/3}\right)$
(C) $(10)^{2/3}$
(D) 5

65. Two lines $\frac{x-3}{1}=\frac{y+1}{3}=\frac{z-6}{-1}$ and $\frac{x+5}{7}=\frac{y-2}{-6}=\frac{z-3}{4}$ intersect at the point R. The reflection f R in the xy – plane has coordinates:

(A) (2, –4, –7)
(B) (2, 4, 7)
(C) (2, –4, 7)
(D) (–2, 4, 7)

66. Contrapositive of the statement
“If two numbers are not equal, then their squares are not equals” is:
(A) If the squares of two numbers are not equal, then the numbers are equal
(B) If the squares of two numbers are equal, then the numbers are not equal
(C) If the squares of two numbers are equal, then the numbers are equal
(D) If the squares of two numbers are not equal, then the numbers are not equal

67. If in a parallelogram ABDC, the coordinates of A, B and C are respectively (1, 2), (3, 4) and (2, 5), then the equation of the diagonal AD is:

(A) $5x-3y+1=0$
(B) $5x+3y-11=0$
(C) $3x-5y+7=0$
(D) $3x+5y-13=0$

68. The integral $\int_{\pi/6}^{\pi/4} \frac{dx}{\sin 2x\left(\tan^5 x + \cot^5 x\right)}$ equals:

(A) $\frac{1}{20}\tan^{-1}\left(\frac{1}{9\sqrt{3}}\right)$

(B) $\frac{1}{10}\left(\frac{\pi}{4} - \tan^{-1}\left(\frac{1}{9\sqrt{l\sqrt{3}}}\right)\right)$

(C) $\frac{\pi}{40}$

(D) $\frac{1}{5}\left(\frac{\pi}{4} - \tan^{-1}\left(\frac{1}{3\sqrt{3}}\right)\right)$

69. Let x, y be positive real numbers and m, n positive integers. The maximum value of the expression $\frac{x^m y^n}{\left(1+x^{2m}\right)\left(1+y^{2n}\right)}$ is:

(A) 1

(B) $\frac{1}{2}$

(C) $\frac{1}{4}$

(D) $\frac{m+n}{6mn}$

70. Let $S_n + 1 + q + q^2 + ... + q^n$ and $T_n = 1 + \left(\frac{q+1}{2}\right) + \left(\frac{q+1}{2}\right)^2 + ... + \left(\frac{q+1}{2}\right)^n$ where q is a real number and $q \neq 1$. If ${}^{101}C_1 + {}^{101}C_2.S_1 + ... + {}^{101}C_{101}.S_{100} = \alpha T_{100}$ then α is equal to:

(A) 2^{99}

(B) 202

(C) 200

(D) 2^{100}

71. Let α and β be the roots of the quadratic equation $x^2 \sin\theta - x\left(\sin\theta\cos\theta + 1\right) + \cos\theta = 0 \left(0 < \theta < 45^0\right)$, and $\alpha < \beta$. Then $\sum_{n=0}^{\infty}\left(\alpha^n + \frac{(-1)^n}{\beta^n}\right)$ is equal to:

(A) $\frac{1}{1-\cos\theta} - \frac{1}{1+\sin\theta}$

(B) $\frac{1}{1+\cos\theta} + \frac{1}{1-\sin\theta}$

(C) $\frac{1}{1-\cos\theta} + \frac{1}{1+\sin\theta}$

(D) $\frac{1}{1+\cos\theta} - \frac{1}{1-\sin\theta}$

72. A bag contains 30 white balls and 10 red balls. 16 balls are drawn one by one randomly from the bag with replacement. If X be the number of white balls drawn, then $\left(\frac{\text{mean of } X}{\text{standard deviation of } X}\right)$ is equal to:

(A) 4

(B) $4\sqrt{3}$

(C) $3\sqrt{2}$

(D) $\frac{4\sqrt{3}}{3}$

73. Let z be a complex number such that $\left|z\right| + z = 3 + i\left(\text{where } i = \sqrt{-1}\right)$. Then $\left|z\right|$ is equal to:

(A) $\frac{\sqrt{34}}{3}$ (B) $\frac{5}{3}$

(C) $\frac{\sqrt{41}}{4}$ (D) $\frac{5}{4}$

74. If $\begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$

$= \left(a+b+c\right)\left(x+a+b+c\right)^2$, x = ≠ and a + b+ +c ≠ 0, then x is equal to:

(A) abc (B) – (a + b + c)

(C) 2(a + b + c) (D) –2 (a + b+ c)

75. Let $\sqrt{3}\hat{i} + j, \hat{i} + \sqrt{3}\hat{j}$ and $\beta\hat{i} + \left(1+\beta\right)\hat{j}$ respectively be the position vectors of the points A, B and C with respect to the origin O. If the distance of C from the bisector of the acute angle between OA and OB is $\frac{3}{\sqrt{2}}$, then the sum of all possible values of β is

(A) 4 (B) 3

(C) 2 (D) 1

76. If 19th terms of non – zero A.P. is zero, then its (49th term) : (29th term) is:

(A) 4:1 (B) 1:3

(C) 3:1 (D) 2:1

77. If $\int \frac{x+1}{\sqrt{2x-1}}dx = f\left(x\right)\sqrt{2x-1} + C$, where C is a constant of integration of integration, then f(x) is equal to:

(A) $\frac{1}{3}\left(x+1\right)$ (B) $\frac{2}{3}\left(x+2\right)$

(C) $\frac{2}{3}\left(x-4\right)$ (D) $\frac{1}{3}\left(x+4\right)$

78. Let a function $f:\left(0,\infty\right) \to \left(0,\infty\right)$ be defined by $f\left(x\right) = \left|1 - \frac{1}{x}\right|$. Then f is:

(A) not injective but it is surjective (B) injective only

(C) neither injective nor surjective (D) both injective as well as surjective

79. Let K be the set of all real values of x where the function $f\left(x\right) = \sin\left|x\right| - \left|x\right| + 2\left(x-\pi\right)\cos\left|x\right|$ is not differentiable. Then the set K is equal to:

(A) ϕ (en empty set) (B) $\{\pi\}$

(C) {0} (D) $\{0, \pi\}$

80. The area (in sq. units) in the first quadrant bounded by the parabola, $y = x^2 + 1$, the tangent to it at the point (2, 5) and the coordinate axes is:

(A) $\frac{8}{3}$ (B) $\frac{37}{24}$

(C) $\frac{187}{24}$ (D) $\frac{14}{3}$

81. Given $\frac{b+c}{11} = \frac{c+a}{12} = \frac{a+b}{13}$ for a ΔABC with usual nation. If $\frac{\cos A}{\alpha} = \frac{\cos \beta}{\beta} = \frac{\cos C}{\gamma}$, then the ordered tried (α, β, γ) has a value:

(A) (7, 19, 25) (B) (3, 4, 5)

(C) (5, 12, 13) (D) (19, 7, 25)

82. The solution of the differential equation $\frac{dy}{dx} = (x - y)^2$ when y(1) = 1, is:

(A) $\log_e \left|\frac{2-x}{2-y}\right| = x - y$ (B) $-\log_e \left|\frac{1-x+y}{}1+x-y\right| = 2(x-1)$

(C) $-\log_e \left|\frac{1+x-y}{1-x+y}\right| = x + y - 2$ (D) $\log_e \left|\frac{2-y}{2-x}\right| = 2(y-1)$

83. Let the length of the latus rectum of an ellipse with its major axis long x – axis and center at the origin, be 8. If the distance between the foci of this ellipse is equal to the length of the length of its minor axis, then which one of the following points lies on it?

(A) $\left(4, \sqrt{2}, 2\sqrt{2}\right)$ (B) $\left(4\sqrt{3}, 2\sqrt{2}\right)$

(C) $\left(4, \sqrt{3}, 2\sqrt{3}\right)$ (D) $\left(4\sqrt{2}, 2\sqrt{3}\right)$

84. let S = {1, 2, … 20}. A subset B of S is said to be "nice", if the sum of the elements of B is 203. Then the probability that a randomly chosen subset of S is 'nice' is:

(A) $\frac{7}{2^{20}}$ (B) $\frac{5}{2^{20}}$

(C) $\frac{4}{2^{20}}$ (D) $\frac{6}{2^{20}}$

85. If the point (2, α, β) lies on the plane which passes through the points (3, 4, 2) and (7, 0, 6) and is perpendicular to the plane $2x - 5y = 15$, then $2\alpha - 3\beta$ is equal to:

(A) 12 (B) 7

(C) 5 (D) 17

86. Let $(x+10)^{50} + (x-10)^{50} = a_0 + a_1x + a_2x^2 + ... + a_{50}x^{50}$, for $x \in R$; then $\frac{a_2}{a_0}$ is equal to:

(A) 12.50 (B) 12.00

(C) 12.25 (D) 12.75

87. The number of functions f from {1, 2, 3, …, 20} only {1, 2, 3, …, 20} such that f(k) is a multiple of 3, whenever k is a multiple of 4, is:

(A) $6^5 \times (15)!$ (B) $5! \times 6!$

(C) $(15)! \times 6!$ (D) $5^6 \times 15$

88. A circle cuts a chord of length 4a on the x – axis and passes through a point on the y – axis, distant 2b from the origin. Then the locus of the center of this circle, is:

(A) a hyperbola (B) an ellipse

(C) a straight line (D) a parabola

89. Let $f(x) = \frac{x}{\sqrt{a^2 + x^2}} - \frac{d - x}{\sqrt{b^2 + (d - x)^2}}, r \in R$ f, where a, b and d are non – zero real constant. Then:

(A) f is an increasing function of x

(B) f is a decreasing function of x

(C) f is not a continuous function of x

(D) f is neither increasing nor decreasing function of x

90. Let A and B be two invertible matrices of order 3 x 3. If det $(ABA^T) = 8$ and $\det(AB^{-1}) = 8$, then det $(BA^{-1} B^T)$ is equal to:

(A) $\frac{1}{4}$ (B) 1

(C) $\frac{1}{16}$ (D) 16

HINTS AND SOLUTIONS

PART A – PHYSICS

1. $\vec{S} = (5\hat{i} + 4\hat{j})2 + \frac{1}{2}(4\hat{j} + 4\hat{j})4$

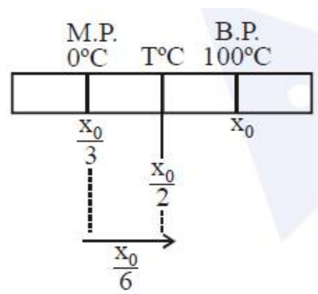
$= 10\hat{i} + 8\hat{j} + 8\hat{i} + 8\hat{j}$

$\vec{r}_2 - \vec{r}_1 = 18\hat{i} + 16\hat{j}$

$\vec{r}_2 = 20\hat{i} + 20\hat{j}$

$|\vec{r}_f| = 20\sqrt{2}$

2. $t = \frac{x_t - x_0}{x_{100} - x_0} 100^{o}C$



$\frac{\frac{x_0}{2} - \frac{x_0}{3}}{x - \frac{x_0}{3}} 100^{o}C$

$= 25°C$

3. $R_g = 20\ \Omega$

$N_L = N_g = N = 30$

FOM = $\frac{1}{\phi}$ = 0.005 A/Div.

Current sentivity = CS = $\left(\frac{1}{0.005}\right) = \frac{\phi}{1}$

$Ig_{max} = 0.005 \times 30$

$= 15 \times 10^{-2} = 0.15$

$15 = 0.15[20 + R]$

$100 = 20 + R$

$R = 80.$

4. $\frac{R_1}{R_2} = \frac{2}{3}$ …(i)

$\frac{R_1 + 10}{R_2} = 1$

$\Rightarrow$ $R_1 + 10 = R_2$ …(ii)

$\frac{2R_2}{3} + 10 = R_2$; $10 = \frac{R_2}{3}$

$\Rightarrow$ $R_2 = 30\ \Omega$ & $R_1 = 20\ \Omega$

$\frac{\frac{30 \times R}{30 + R}}{30} = \frac{2}{3}$

$R = 60\ \Omega$

5. $I = \frac{MR^2}{2} + 2\left(\frac{MR^2}{4} + MR^2\right)$

$= \frac{MR^2}{2} + \frac{MR^2}{2} + 2MR^2 = 3\ MR^2$

6. $2.5 = 1 \times 5 \sin\theta$

$\sin\theta = 0.5 = \frac{1}{2}$

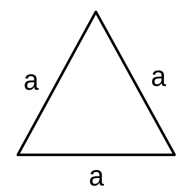
$\theta = \frac{\pi}{6}$

7. Total length L will remain constant

L = (3a) N (N = total turns)

And length of winding = (d) N = ℓ

(d = diameter of wire)



Self inductance = $\mu_0 n^2 A\ell$

$= \mu_0 n^2 \left(\frac{\sqrt{3}\ a^2}{4}\right) dN$

$\propto a^2 N \propto a$

So self inductance will become 3 times.

8. $\frac{dp}{dt} = F = kt$

$\int_P^{3P} dP = \int_0^T kt\ dt$

$2p = \frac{KT^2}{2}$; $T = 2\sqrt{\frac{p}{k}}$

9. $\chi = \frac{I}{H}$

$I = \frac{\text{Magnetic moment}}{\text{Volume}}$

$I = \frac{20 \times 10^{-6}}{10^{-6}} = 20\ N/m^2$

$\chi = \frac{20}{60 \times 10^3} = \frac{1}{3} \times 10^{-3}$

$= 0.33 \times 10^{-3} = 3.3 \times 10^{-4}$

10. Angular frequency of pendulum

$\omega \propto \sqrt{\frac{g_{eff}}{\ell}}$

$\therefore \frac{\Delta\omega}{\omega} = \frac{1}{2}\frac{\Delta g_{eff}}{g_{eff}}$

$$\Delta\omega = \frac{1}{2}\frac{\Delta g}{g}\times\omega$$

[ω_s = angular frequency of support]

$$\frac{\Delta\omega}{\omega} = \frac{1}{2}\times\frac{\Delta g}{g}$$

$$= \frac{1}{2}\times\frac{2(A\omega_5^s)}{10}$$

$$\Rightarrow \frac{\Delta\omega}{\omega} = \frac{1\times10^{-2}}{10} = 10^{-3}$$

11. $I = \frac{6}{300} = 0.002$ (D_2 is in reverse bias)

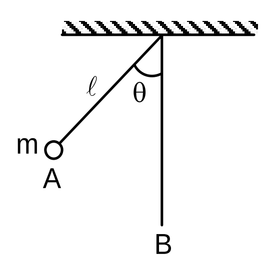
12. $U = -\vec{P}\cdot\vec{E}$
$= -PE\cos\theta$
$= -(10^{-29})(10^{3})\cos 45°$
$= -0.707\times10^{-26}$ J
$= -7\times10^{-27}$ J

13. $0.1\times400\times(500-T) = 0.5\times4200\times(T-30) + 800(T-30)$
$\Rightarrow$ $40(500-T) = (T-30)(2100+800)$
$\Rightarrow$ $20000 - 40T = 2900T - 30\times2900$
$\Rightarrow$ $20000 + 30\times2900 = T(2940)$
$T = 30.4°C$

$$\frac{\Delta T}{T}\times100 = \frac{6.4}{30}\times100 \simeq 20\%$$

14. BONUS

15. Maximum kinetic energy at lowest point B is given by



$K = mg\ell(1-\cos\theta)$

where θ = angular amp.

$K_1 = mg\ell(1-\cos\theta)$

$K_2 = mg(2\ell)(1-\cos\theta)$

$K_2 = 2K_1$

16. $\Delta\ell_1 = \Delta\ell_2$

$\ell\alpha_1\Delta T_1 = \ell\alpha_2\Delta T_2$

$$\frac{\alpha_1}{\alpha_2} = \frac{\Delta T_1}{\Delta T_2} \quad ; \quad \frac{4}{3} = \frac{T-30}{180-30}$$

$T = 230°C$

17. $\frac{F}{A} = y \cdot \frac{\Delta \ell}{\ell}$; $[Y] = \frac{F}{A}$

Now from dimension

$$F = \frac{ML}{T^2} \quad ; \quad L = \frac{F}{M} \cdot T^2$$

$$L^2 = \frac{F^2}{M^2}\left(\frac{V}{A}\right)^4 \qquad \because \quad T = \frac{V}{A}$$

$$L^2 = \frac{F^2}{M^2 A^2}\,\frac{v^4}{A^2} \qquad F = MA$$

$$L^2 = \frac{V^4}{A^2}$$

$$[Y] = \frac{[F]}{[A]} = F^1 V^{-4} A^2$$

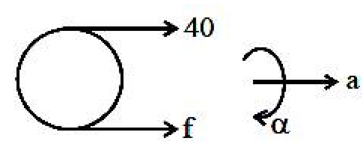
18. $40 + f = m(R\alpha)$ ...(i)

$40 \times R - f \times R = mR{=}2^{\alpha}$

$40 - f = mR\alpha$ ...(ii)

From (i) and (ii)

$$\alpha = \frac{40}{mR} = 16$$



19. Intensity of EM wave is given by

$$I = \frac{\text{Power}}{\text{Area}} = \frac{1}{2}\varepsilon_0 E_0^2 C$$

$$= \frac{27 \times 10^{-3}}{10 \times 10^{-6}} = \frac{1}{2} \times 9 \times 10^{-2} \times E^2 \times 3 \times 10^8$$

$$E = \sqrt{2} \times 10^3 \text{ kV / m}$$

$$= 1.4 \text{ kV/m}$$

20. Potential difference across AB will be equal to battery equivalent across CD.

$$V_{AB} = V_{CD} = \frac{\frac{E_1}{r_1} + \frac{E_2}{r_2} + \frac{E_3}{r_3}}{\frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3}} = \frac{\frac{1}{1} + \frac{2}{1} + \frac{3}{1}}{\frac{1}{1} + \frac{1}{1} + \frac{1}{1}}$$

$$= \frac{6}{3} = 2V$$

21. $\because \quad g = \frac{GM}{R^2}$

$$\frac{g_p}{g_e} = \frac{M_p}{M_e}\left(\frac{R_e}{R_p}\right)^2 = 3\left(\frac{1}{3}\right)^2 = \frac{1}{3}$$

Also, $T \propto \frac{1}{\sqrt{g}}$

$\Rightarrow \quad \frac{T_p}{T_e} = \sqrt{\frac{g_e}{g_p}} = \sqrt{3}$

$\Rightarrow \quad T_p = 2\sqrt{3}s$

22. Analysis of graph says

(1) Amplitude varies as 8 – 10 V or 9 ± 1

(2) Two time period 100 μs (signal wave) and 8 μs (carrier wave)

Hence signal is $\left[9 \pm 1 \sin\left(\frac{2\pi t}{T_1}\right)\right] \sin\left(\frac{2\pi t}{T_2}\right)$

$= 9 \pm 1 \sin (2\pi \times 10^4 t) \sin 2.5\pi \times 10^5 t$

23. VT = K

$\Rightarrow \quad V\left(\frac{PV}{nR}\right) = k$

$\Rightarrow \quad PV^2 = K$

$\because \quad C = \frac{R}{1-x} + C_V$ (For polytropic process)

$C = \frac{R}{1-2} + \frac{3R}{2} = \frac{R}{2}$

$\therefore \quad \Delta Q = nC\,\Delta T$

24. $100 \times S_A \times [100 - 90] = 50 \times S_B \times (90 - 75)$

$2S_A = 1.5\, S_B$

$S_A = \frac{3}{4} S_B$

Now, $100 \times S_A \times [100 - T] = 50 \times S_B (T - 50)$

$2 \times \left(\frac{3}{4}\right)(100 - T) = (T - 50)$

$300 - 3T = 2T - 100$

$400 = 5T$

$T = 80$

25. For $M \to L$ steel

$\frac{1}{\lambda} = K\left(\frac{1}{2^2} - \frac{1}{3^2}\right) = \frac{K \times 5}{36}$

for $N \to L$

$\frac{1}{\lambda'} = K\left(\frac{1}{2^2} - \frac{1}{4^2}\right) = \frac{K \times 3}{16}$

$\lambda' = \frac{20}{27}\lambda$

26. i = e

$r_1 = r_2 = \frac{A}{2} = 30^0$

by Snell's law

$$1 \times \sin i = \sqrt{3} \times \frac{1}{2} = \frac{\sqrt{3}}{2}$$

i = 60

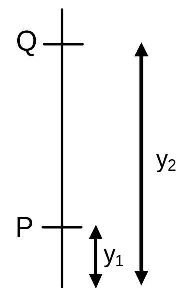
27. For diffraction

Location of 1st minima

$$y_1 = \frac{D\lambda}{a} = 0.2469D\lambda$$

Location of 2nd minima

$$y_2 = \frac{2D\lambda}{a} = 0.4938D\lambda$$



Now for interference

Path for interference

Path difference at P.

$$\frac{dy}{D} = 4.8\,\lambda$$

Path difference at P.

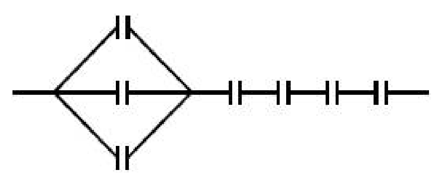
$$\frac{dy}{D} = 9.6\lambda$$

So orders of maxima in between P and Q is

5, 6, 7, 8, 9

So 5 bright fringes all present between P & Q.

28. $C_{eq} = \frac{6}{13}\mu F$



Therefore three capacitors most be in parallel to get 6 in

$$\frac{1}{C_{eq}} = \frac{1}{3C} + \frac{1}{C} + \frac{1}{C} + \frac{1}{C} + \frac{1}{C}$$

$$C_{eq} = \frac{3C}{13} = \frac{6}{13}\mu F$$

29. $\vec{F}_{net} = d\vec{E} + q(\vec{v} \times \vec{B})$

$= (2q\hat{i} + 3q\hat{i}) + q(\vec{v} \times \vec{B})$

W $= \vec{F}_{net} \cdot \vec{S}$

= 2q + 3q

= 5q

30. $\frac{hc}{\lambda_1} = \phi + eV_1$ ...(i)

$\frac{hc}{\lambda_2} = \phi + eV_2$ ...(ii)

(i) – (ii)

$$hc\left(\frac{1}{\lambda_1}-\frac{1}{\lambda_2}\right)=e(V_1-V_2)$$

$$\Rightarrow \quad V_1-V_2=\frac{hc}{e}\left(\frac{\lambda_2-\lambda_1}{\lambda_1-\lambda_2}\right)$$

$$= (1240\text{ nm V})\ \frac{100\text{ nm}}{300\text{ nm}\times 400\text{ nm}}$$

$$= \frac{12.4}{12} \approx 1\text{ V.}$$

PART B – CHEMISTRY

31. In order to be spontaneous ΔG^O should be –ve
$\Delta G^O = \Delta H^O - T\Delta S^O$
$0 = 491.1 \times 10^3 - T \times 198$
$T=\frac{491100}{198}=2480$
If temp is above 2480 K, the reaction will be spontaneous.

32. Fact based, go through definition.

33. Th is a metal having large size and oxalate is a bidentate ligand hence its co-ordination number in given complex is 10.

34. Since $LiAlH_4$ is a strong reducing agent, it will reduce $-NO_2$, $-\overset{\overset{\displaystyle O}{\|}}{C}-O-H$ and ketonic group but cannot reduce normal alkene >C=C<

35. $\Delta G^O = \Delta H^O - T\Delta S^O$
$\Delta G^O = A - BT$
In endothermic reaction ΔH = +ve. Hence, A = +ve

36. $\sqrt{3}a = 4R$

$R=\frac{\sqrt{3}a}{4}$

$2(R + r) = a$

$2\left(\frac{\sqrt{3}a}{4}+r\right)=a$

$\frac{\sqrt{3}a}{2}+2r=a$

$2r=a-\frac{\sqrt{3}a}{2}=\frac{2a-\sqrt{3}a}{2}$

$r=\frac{2a-1.7329}{4}=\frac{.268a}{4}$ = .067a

37. SiH_4 has complete octet hence it is not an electron deficient hydride.

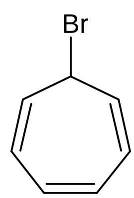
38. $\Delta G^o = -nFE^o_{cell} = -2.303RT \log K_{eq}$

$2F\,E^o_{cell} = 2.303RT \log K_{eq}$

$2\,E^o_{cell} = \frac{2.303RT}{F} \log 10 \times 10^{15}$

$E^o_{cell} = \frac{.059}{2} \times 16 = 8 \times .059 \Rightarrow .472$

39. Electronegativity increases from left to right in a period and decreases down the group.

40. On ionization  , this compound will produce Aromatic cation, which is stable.

41. $\lambda = \frac{h}{mv}$

According to Einstein's theory of photoelectric effect:

$h\nu = h\nu_o + KE$

$h\nu = h\nu_o + \frac{1}{2}mv^2$

$2h(\nu - \nu_o) = mv^2$

$\frac{2h(\nu - \nu_o)}{m} = v^2$

$v \propto (\nu - \nu_o)^{\frac{1}{2}}$

$\lambda \propto \frac{h}{m(\nu - \nu_o)^{\frac{1}{2}}}$

$\lambda \propto \frac{1}{(\nu - \nu_o)^{\frac{1}{2}}}$

42. Option B and option D both will react with Grignard reagent and decolourizes Br_2/H_2O. (IIT has given option D only)

43. After protonation at b or c or d the conjugate acid is stabilized by resonance.

44. Acid rain reacts with marble. Hence, The Taj Mahal which made up of marble is discoloured.

45. Inert pair effect gradually increases down the group. Hence, stability of lower oxidation state increases down the group.

46. $\Delta G^o = -2.303 RT \log K_{eq}$

$= -2.303 \times 8.314 \times 298 \log 10^{-14}$
$= -2.303 \times 8.314 \times 298 \times -14$
$= 79,881.87$
$\simeq 80$ KJ mol^{-1}

47. Calcination takes place in absence of air. Hence step 2 is not defining it.

48. Br_2/NaOH converts amide into primary amine having one carbon atom less, which gives carbylamine test.

49. Go through different types of colloid and their examples.

50. 4-hydroxy butanoic acid undergoes intermolecular esterification to give polymer.

51. $\underset{1-\alpha}{K_2[HgI_4]} \rightleftharpoons \underset{2\alpha}{2K^+} + \underset{\alpha}{[HgI_4]^{2-}}$

Total number of particle = $1 + 2\alpha$

Hence, Van't Hoff factor $= \frac{1+2\alpha}{1}$

$= \frac{1+2\times 0.4}{1} = 1 + 0.8 \Rightarrow 1.8$

52. Apply law of equivalence:

$25 \times N = 30 \times 0.1 \times 2$

$N_{HCl} = \frac{30\times 0.2}{25} = \frac{6}{5}\times 0.2 = \frac{1.2}{5}$

For the 2nd titration

$\frac{1.2}{5} \times V_{HCl} = 30 \times 0.2$

$V_{HCl} = \frac{6\times 5}{1.2} = \frac{30}{1.2}$ = 25 ml

53. For zero order reaction

$C_o - C_t = Kt$
$0.5M - 0.2M = Kt$
$0.3 = Kt$ ----- (1)

'K' can be calculated by

$t_{1/2} = \frac{C_o}{2K}$

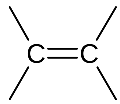
$6 = \frac{0.2}{2K}$

$K = \frac{0.2}{12} = \frac{2\times 10^{-1}}{12} = \frac{1}{60}$

Putting the value of K in eq (1)

$t = \frac{0.3}{K} = \frac{0.3}{1/60} = 60 \times 0.3 = 18 \text{Hr}$

54. Fact based

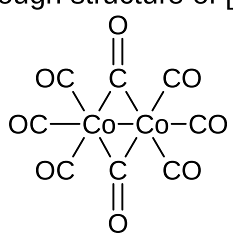
55. Attack on  is more preferred than benzene ring.

56. First Markonikov's addition one alkene followed by intramolecular Friedel-Craft alkylation takes place.

57. SO_2 gas causes stiffness of flower buds?

58. Go through structure of amino acids.

59. Go through structure of $[Co_2(CO)_8]$



60. $MnO_2 \xrightarrow{4KOH,\ O_2} \underset{(Green)}{2K_2MnO_4} + 2H_2O$

$3K_2MnO_4 \xrightarrow{4\ HCl} \underset{(Purple)}{2KMnO_4} + MnO_2 + 2H_2O$

$2KMnO_4 \xrightarrow{H_2O,\ KI} 2MnO_2 + 2KOH + KIO_3$

PART C – MATHEMATICS

61. $\dfrac{x\cos 4x \sin^2 2x}{\sin^2 x.\cos^2 2x.\sin 4x}$

$= \dfrac{4x}{\sin 4x}.\dfrac{\cos 4x}{\cos^2 2x}\cos^2 x$

$\Rightarrow 1 \text{ as } x \to 0$

62. $\left(\cot^{-1}(x)-5\right)\left(\cot^{-1}(x)-2\right) > 0$

$\Rightarrow \cot^{-1}(x) \in (-\infty, 2) \cup (5, \infty)$

Put $0 < \cot^{-1}(x) < \pi$

$\Rightarrow \cot^{-1}(x) \Rightarrow (0, 2)$

$\Rightarrow x \in (\cot 2, \infty)$

63. $2b = 5$ and $2ae = 13$

$(ae)^2 = a^2 + b^2$

$\Rightarrow a^2 = (ae)^2 - b^2 = \dfrac{169}{4} - \dfrac{25}{4}$

$\Rightarrow a = 6$

$e = \frac{ae}{a} = \frac{13}{12}$

64. $y^2 = -4\left(x - a^2\right)$

Vertices of triangle are $\left(a^2, 0\right)$ and (0, 2a) and (0, –2a)

Area $= \frac{1}{2}\left(a^2\right)\left(4a\right) = 250$

$\Rightarrow a^3 = 125$

65. Points on the given lines are $\left(\lambda + 3,\ 3\lambda - 1,\ -\lambda + 6\right)$ and $\left(7\alpha - 5,\ -6\alpha + 2,\ 4\alpha + 3\right)$

$\Rightarrow \lambda + 3 = 7\alpha - 5$

$3\lambda - 1 = -6\alpha + 2$

$\Rightarrow \alpha = 1,\ \lambda = -1$

Point R is (2, –4, 7)

Image of R under xy – plane is (2, –4, –7)

66. Contra positive of $p \to q$ is $\sim q \to \sim p$

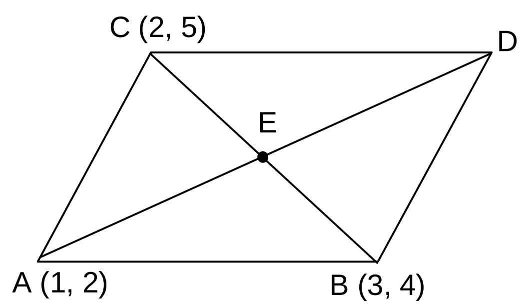
$\therefore$ Answer is C

67. E is $\left(\frac{5}{2}, \frac{9}{2}\right)$

Slope of $AD = \frac{5}{3}$

Equation of AD is $y - 2 = \frac{5}{3}\left(x - 1\right)$

$\Rightarrow 5x - 3y + 1 = 0$



68. $I = \int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \frac{\sec^2 x\,dx}{2\tan x\left(\tan^5 x + \cot^5 x\right)}$

Put $\tan x = t$

$= \int_{\frac{1}{\sqrt{3}}}^{1} \frac{dt}{2t\left(t^5 + \frac{1}{t^5}\right)}$

$= \int_{\frac{1}{\sqrt{3}}}^{1} \frac{t^4 dt}{2\left(t^{10} + 1\right)}$

Put $t^5 = y$

$I = \frac{1}{10}\tan^{-1}\left(y\right)\Big]_{3^{-\frac{5}{2}}}^{1}$

$= \frac{1}{10}\left(\frac{\pi}{4} - \tan^{-1}\left(\frac{1}{3^{5/2}}\right)\right)$

69. $\frac{x^m y^n}{(1+x^{2m})(1+y^{2n})}$

$$=\frac{1}{\left(x^m+\frac{1}{x^m}\right)\left(y^n+\frac{1}{y^n}\right)}$$

Put $x^m+\frac{1}{x^m}\geq 2$

$$\Rightarrow \frac{1}{\left(x^m+\frac{1}{x^m}\right)}\leq\frac{1}{2}$$

$\Rightarrow$ Maximum Value $=\frac{1}{4}$

70. $\sum_{r=1}^{101} {}^{101}C_r\, s_{r-1}$

$$=\sum_{r=1}^{101} {}^{101}C_r \frac{q^r-1}{q-1}$$

$$=\frac{1}{q-1}\left(\sum_{r=1}^{101} {}^{101}C_r\, q^r-\sum_{r=1}^{101} {}^{101}C_r\right)$$

$$=\frac{1}{q-1}\left((1+q)^{101}-1-2^{101}+1\right)$$

$$=\frac{\alpha}{2^{100}}\left(\frac{(1+q)^{101}-2^{101}}{q-1}\right)$$

$\Rightarrow \alpha=2^{100}$

71. Using quadratic formula,

$$x=\frac{(\cos\theta\sin\theta+1)\pm\sqrt{(\cos\theta\sin\theta+1)^2-4\sin\theta\cos\theta}}{2\sin\theta}$$

$$=\frac{(\cos\theta\sin\theta+1)^2\pm(\cos\theta\sin\theta-1)}{2\sin\theta}$$

$=\cos\theta,\ \operatorname{cosec}\theta$

$\alpha=\cos\theta,\ \beta=\operatorname{cosec}\theta$

$$\therefore \sum_{n=0}^{\infty}\alpha^n+\frac{(-1)^n}{\beta^n}$$

$$=\sum_{n=0}^{\infty}(\operatorname{cosec})^n+\sum_{n=0}^{\infty}(-\sin\theta)^n$$

$$=\frac{1}{1-\cos\theta}+\frac{1}{1+\sin\theta}$$

$\therefore$ (C) is the correct answer.

72. There are 30 white balls and 10 red balls

P (white ball) $= \frac{30}{40} = \frac{3}{4} = p$

$\Rightarrow q = \frac{1}{4}$

$$\frac{\text{mean}(x)}{\text{standard deviation}(x)} = \frac{np}{\sqrt{npq}}$$

$$= \sqrt{\frac{np}{q}} = \sqrt{\frac{16 \times \left(\frac{3}{4}\right)}{\frac{1}{4}}} = 4\sqrt{3}$$

73. $z = x + iy$

$\sqrt{x^2 + y^2} + x + iy = 3 + i$

$\Rightarrow y = 1$

$\sqrt{x^2 + 1} + x = 3 \Rightarrow x^2 + 1 = 9 - 6x + x^2$

$\Rightarrow x = \frac{4}{3}$

$|z| = \sqrt{x^2 + y^2} = \frac{5}{3}$

74. $R_1 \to R_1 + R_2 + R_3$

$$(a+b+c)\begin{vmatrix} 1 & 1 & 1 \\ 2b & b-a-c & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$$

$c_3 \to c_3 - c_1,\ c_2 \to c_2 - c_1$

$$= (a+b+c)\begin{vmatrix} 1 & 0 & 0 \\ 2b & -(a+b+c) & 0 \\ 2c & 0 & -(a+b+c) \end{vmatrix}$$

$= (a+b+c)^3$

$= (a+b+c)(x+a+b+c)^2$

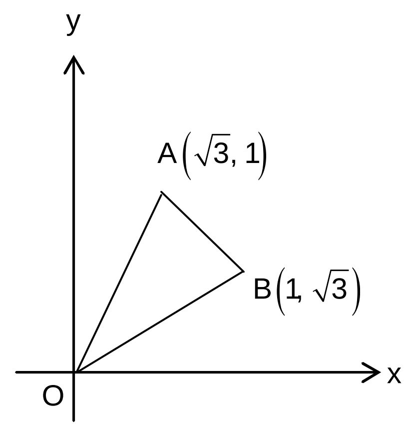
75. Equation of angle bisector of DA and OB is
$y = x$

Given that, $\left|\frac{\beta-(1-\beta)}{\sqrt{2}}\right| = \frac{3}{\sqrt{2}}$

$2\beta - 1 = \pm 3$

$\Rightarrow \beta = 2, -1$

Sum of values of $\beta = 1$



76. $a + 18d = 0 \Rightarrow a = -18d$

$\frac{t_{49}}{t_{29}} = \frac{a+48d}{a+28d} = \frac{-18d+48d}{-18d+28d}$

$= \frac{30d}{10d} = 3$

77. Put $2x - 1 = t^2$

$\Rightarrow \int \frac{x+1}{\sqrt{2x-1}} dx$

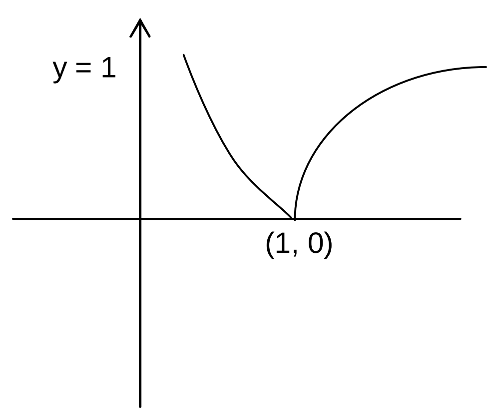
$= \int \left(\frac{t^2+3}{2}\right) dt = \frac{t^3}{6} + \frac{3t}{2} + C$

$= t\left(\frac{t^2}{6} + \frac{3}{2}\right) + C$

$= \sqrt{2x-1}\left(\frac{x+4}{3}\right) + C$

78. $y = \left|1 - \frac{1}{x}\right|$

Neither one - one nor Onto



79. At $x = \pi +$ or $x = \pi -$

$f(x) = \sin x - x + 2(x-\pi)\cos x$

$f(x)$ is differentiable

For $x = 0+$

$f(x) = \sin x - x + 2(x-\pi)\cos x$

$$f'(x) = \cos x - 1 + 2\cos x - 2(x - \pi)\sin x$$
$$f'(0+) = 2$$
For $x = 0-$
$$f(x) = -\sin x + x + 2(x - \pi)\cos x$$
$$f'(x) = -\cos x + 1 + 2\cos x - 2(x - \pi)\sin x$$
$$f'(0-) = 2$$
LHD = RHD
$\therefore$ f is differentiable at x = 0.

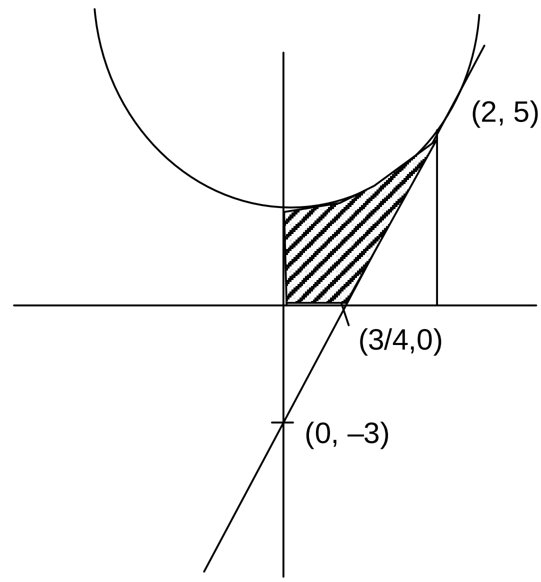
80. Equation of tangent at (2, 5) is $\frac{y+5}{2} = x(2) + 1$

or $y = 4x - 3$

Required Area $= \int_0^2 (x^2 + 1)dx - \frac{1}{2}\left(2 - \frac{3}{4}\right).(5)$

$= \frac{x^3}{3} + x\Big]_0^2 - \frac{25}{8}$

$= \frac{8}{3} - \frac{9}{8} = \frac{37}{24}$



81. $\frac{b+c}{11} = \frac{c+a}{12} = \frac{a+b}{13} = \frac{a+b+c}{18}$

$\Rightarrow a = 7k, b = 6k, c = 5k$

$\cos A = \frac{b^2 + c^2 - a^2}{2bc} = \frac{1}{5}$

$\cos B = \frac{19}{25}, \cos C = \frac{5}{7}$

$\frac{1}{5\alpha} = \frac{19}{35\beta} = \frac{5}{7\gamma}$

$\Rightarrow \frac{7}{35\alpha} = \frac{19}{35\beta} = \frac{25}{35\gamma}$

$\alpha : \beta : \gamma = 7 : 19 : 25$

82. $u = x - y$

$\frac{du}{dx} = 1 - \frac{dy}{dx}$

$\Rightarrow 1 - \frac{du}{dx} = u^2$

$1 - u^2 = \frac{du}{dx}$

$\frac{du}{1-u^2} = dx$

$\Rightarrow \frac{1}{2}\log\left|\frac{1+u}{1-u}\right| = x + c$

$\Rightarrow \frac{1}{2}\log\left|\frac{1+x-y}{1-x+y}\right| = x + c$

Put $x = 1 \Rightarrow \quad c = -1$

$\Rightarrow \log\left|\frac{1+x-y}{1-x+y}\right| = 2(x-1)$

83. Consider $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

Given that $2b = 2ae$

$\Rightarrow b = ae$ and $\frac{2b^2}{a} = 8$

$a(1-e^2) = 4,\ a^2e^2 = a^2(1-e^2)$

$\Rightarrow e^2 = \frac{1}{2}$

$\Rightarrow a = 8,\ b = 4\sqrt{2}$

Hence equation of ellipse is $\frac{x^2}{64} + \frac{y^2}{32} = 1$

$(4\sqrt{3},\ 2\sqrt{2})$ lies on this

84. Sum of all elements of S = 210

So X be a nice set if $x = \{S-\{7\},\ S-\{1, 6\},\ S-\{2, 5\}, S-\{3, 4\},\ S-\{1, 2, 4\}\}$

$P(x) = \frac{5}{2^{20}}$

$\therefore$ (2) is the answer.

85. $A(7, 0, 6)$ and B (3, 4, 2)

$\overrightarrow{AB} = -4\hat{i} + 4\hat{j} - 4\hat{k}$

Also $2\hat{i} - 5\hat{j}$ is parallel to the plane

$\Rightarrow$ Normal perpendicular to the required plane is $\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 1 \\ 2 & -5 & 0 \end{vmatrix} = 5\hat{i} + 2\hat{j} - 3\hat{k}$

Equation of the plane is $5(x-7) + 2(y-0) - 3(z-6) = 0$

$5x + 2y - 3z = 17$

$(2, \alpha, \beta)$ lies on this

$10 + 2\alpha - 3\beta = 17$

$\Rightarrow 2\alpha - 3\beta = 7$

86. $(10+x)^{50}+(10-x)^{50}$

$a_0=\left(10^{50}\right)(2)$

$a_2={}^{50}C_2\,(10)^{48}\,(2)$

$\frac{a_2}{a_0}=\frac{{}^{50}C_2\,(10)^{48}\,(2)}{10^{52}(2)}=12.25$

87. $k=\{4,8,12,16,20\}$

$f(k)$ can takes the values $\{3,6,9,12,15,18\}$

Number of ways $={}^{6}C_5\,.5!$

$\therefore$ Total number of onto functions

$={}^{6}C_5\,.5!(15!)$

$=(6!)(15!)$

88. $k^2+4a^2=r^2$ and

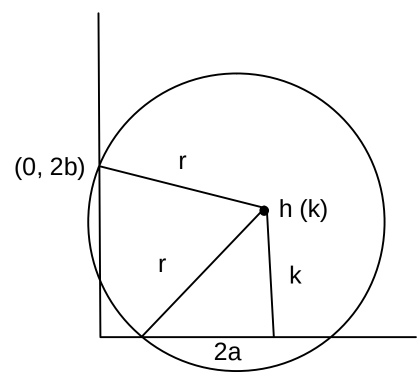
$(h-0)^2+(k-2b)^2=r^2$

$\Rightarrow h^2+(k-2b)^2=k^2+4a^2$

$\Rightarrow h^2=4bk-4b^2+4a^2$

Locus is $x^2=4\left(by-b^2+a^2\right)$

$\therefore$ Parabola.



89. $f(x)=x\left(x^3+a^2\right)^{\frac{-1}{2}}-(d-x)\left(b^2+(d-x)^2\right)^{-\frac{1}{2}}$

$f'(x)=\frac{a^2}{\left(x^2+a^2\right)\sqrt{x^2+a^2}}+\frac{b^2}{\left(b^2+(d-x)^2\right)\sqrt{b^2+(d-x)^2}}$

$=+ve$

90. $\left|ABA^T\right|=|A|.|B|.\left|A^T\right|=|A|^2\,|B|$

$\left|AB^{-1}\right|=8\Rightarrow|A|=8|B|$

$\left|BA^{-1}B\right|=\frac{|B|^2}{|A|}=\frac{|B|^2}{8|B|}=\frac{|B|}{8}=\frac{1}{16}$