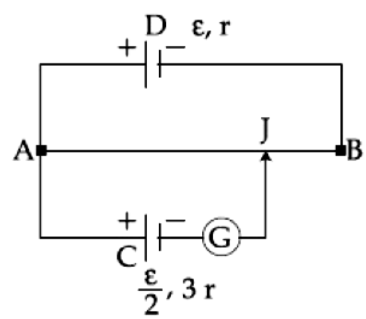
PART –A (PHYSICS)

1. In an electron microscope, the resolution that can be achieved is of the order of the wavelength of electrons used. To resolve a width of 7.5×10^{-12} m, the minimum electron energy required is close to:
(A) 500 keV (B) 100 keV
(C) 1 keV (D) 25 keV

2. To mop-clean a floor, a cleaning machine presses a circular mop of radius R vertically down with a total force F and rotates it with a constant angular speed about its axis. If the force F is distributed uniformly over the mop and the floor is μ, the torque, applied by the machine on the mop is:
(A) $\mu FR/3$ (B) $\mu FR/6$
(C) $\mu FR/2$ (D) $\frac{2}{3}\mu FR$

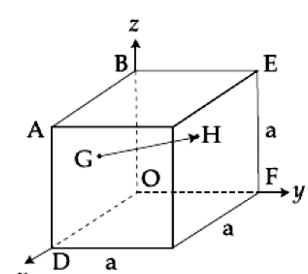
3. A potentiometer wire AB having length L and resistance 12 r is joined to a cell D of emf ε and internal resistance r. A cell C having emf $\varepsilon/2$ and internal resistance 3r is connected. The length AJ at which the galvanometer as shown in figure shows no deflection is:



(A) $\frac{11}{12}L$ (B) $\frac{11}{24}L$
(C) $\frac{13}{24}L$ (D) $\frac{5}{12}L$

4. A TV transmission tower has a height of 140 m and the height of the receiving antenna is 40 m. What is the maximum distance upto which signals can be broadcasted from this tower in LOS (Line of Sight) mode? (Given : radius of earth = 6.4×10^6 m).
(A) 65 km (B) 48 km
(C) 80 km (D) 40 km

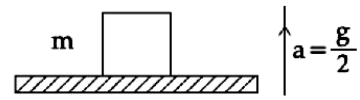
5. In the cube of side 'a' shown in the figure, the vector from the central point of the face ABOD to the central point of the face BEFO will be:



(A) $\frac{1}{2}a(\hat{k}-\hat{i})$ (B) $\frac{1}{2}a(\hat{i}-\hat{k})$
(C) $\frac{1}{2}a(\hat{j}-\hat{i})$ (D) $\frac{1}{2}a(\hat{j}-\hat{k})$

6. A uniform metallic wire has a resistance of 18 Ω and is bent into an equilateral triangle. Then, the resistance between any two vertices of the triangle is:
(A) 4 Ω (B) 8 Ω
(C) 12 Ω (D) 2 Ω

7. A block of mass m is kept on a platform which starts from rest with constant acceleration g/2 upward, as shown in figure. Work done by normal reaction on block in time t is:



(A) $-\frac{mg^2t^2}{8}$ (B) $\frac{mg^2t^2}{8}$
(C) 0 (D) $\frac{3mg^2t^2}{8}$

8. In a Young's double slit experiment with slit separation 0.1 mm, one observes a bright fringe at angle $\frac{1}{40}$ rad by using light of wavelength λ_1. When the light of wavelength λ_2 is used a bright fringe is seen at the same angle in the same set up. Given that λ_1 and λ_2 are in visible range (380 nm to 740 nm), their values are:
(A) 625 nm, 500 nm (B) 380 nm, 525 nm
(C) 380 nm, 500 nm (D) 400 nm, 500 nm

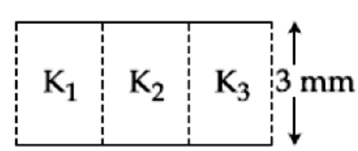
9. Two guns A and B can fire bullets at speed 1 km/s and 2 km/s respectively. From a point on a horizontal ground, they are fired in all possible directions. The ratio of maximum areas covered by the bullets fired by the two guns, on the ground is:
(A) 1 : 16 (B) 1 : 2
(C) 1 : 4 (D) 1 : 8

10. The density of a material in SI units is 128 kg m^{-3}. In certain units in which the unit of length is 25 cm and the unit of mass 50 g, the numerical value of density of the material is:
(A) 40 (B) 16
(C) 640 (D) 410

11. A magnet of total magnetic moment $10^{-2}\hat{i}$ A-m^2 is placed in a time varying magnetic field, $B\hat{i}(\cos\omega t)$ where B = 1 Tesla and ω = 0.125 rad/s. The work done for reversing the direction of the magnetic moment at t = 1 second, is:
(A) 0.01 J (B) 0.007 J
(C) 0.028 J (D) 0.014 J

12. A heat source at T = 10^3 K is connected to another heat reservoir at T = 10^2 K by a copper slab which is 1 m thick. Given that the thermal conductivity of copper is 0.1 WK^{-1} m^{-1}, the energy flux through it in the steady state is:
(A) 90 Wm^{-2} (B) 120 Wm^{-2}
(C) 65 Wm^{-2} (D) 200 Wm^{-2}

13. A parallel plate capacitor is of area 6 cm^2 and a separation 3 mm. The gap is filled with three dielectric materials of equal thickness (see figure) with dielectric constants K_1 = 10, K_2 = 12 and K_3 = 14. The dielectric constant of a material which when fully inserted in above capacitor, gives same capacitance would be:



(A) 4 (B) 14
(C) 12 (D) 36

14. A charge Q is distributed over three concentric spherical shell of radii a, b, c (a < b < c) such that their surface charge densities are equal to one another.
The total potential at a point at distance r from their common centre, where r < a, would be:

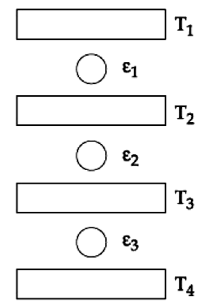
(A) $\frac{Q}{12\pi \in_0}\frac{ab+bc+ca}{abc}$

(B) $\frac{Q\left(a^2+b^2+c^2\right)}{4\pi \in_0 \left(a^3+b^3+c^3\right)}$

(C) $\frac{Q}{4\pi \in_0 \left(a+b+c\right)}$

(D) $\frac{Q\left(a+b+C\right)}{4\pi \in_0 \left(a^2+b^2+c^2\right)}$

15. Three Carnot engines operate in series between a heat source at a temperature T_1 and a heat sink at temperature T_4 (see figure). There are two other reservoirs at temperature T_2 and T_3, as shown, with $T_1 > T_2 > T_3 > T_4$. The three engines are equally efficient if:



(A) $T_2 = \left(T_1T_4\right)^{1/2}; T_3 = \left(T_1^2T_4\right)^{1/3}$

(B) $T_2 = \left(T_1^2T_4\right)^{1/3}; T_3 = \left(T_1T_2^4\right)^{1/3}$

(C) $T_2 = \left(T_1T_4^2\right)^{1/3}; T_3 = \left(T_1^2T_4\right)^{1/3}$

(D) $T_2 = \left(T_1^3T_4\right)^{1/4}; T_3 = \left(T_1T_4^3\right)^{1/4}$

16. A satellite is moving with a constant speed υ in circular orbit around the earth. An object of mass 'm' is ejected from the satellite such that it just escapes from the gravitational pull of the earth. At the time of ejection, the kinetic energy of the object is:

(A) $2\ m\upsilon^2$

(B) $m\upsilon^2$

(C) $\frac{1}{2}m\upsilon^2$

(D) $\frac{3}{2}m\upsilon^2$

17. Water flows into a large tank with flat bottom at the rate of $10^{-4}\ m^3s^{-1}$. Water is also leaking out of a hole of area 1 cm^2 at its bottom. If the height of the water in the tank remains steady, then this height is:

(A) 5.1 cm
(B) 1.7 cm
(C) 4 cm
(D) 2.9 cm

18. A string of length 1 m and mass 5 g is fixed at both ends. The tension in the string is 8.0 N. The string is set into vibration using an external vibrator of frequency 100 Hz. The separation between successive nodes on the string is close to:

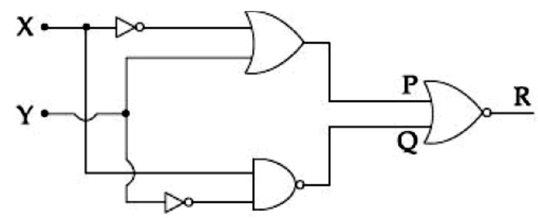
(A) 10.0 cm
(B) 33.3 cm
(C) 16.6 cm
(D) 20.0 cm

19. A train moves towards a stationary observer with speed 34 m/s. The train sounds a whistle and its frequency registered by the observer is f_1. If the speed of the train is reduced to 17 m/s, the frequency registered is f_2. If speed of sound is 340 m/s, then the ratio f_1/f_2 is:

(A) 18/17
(B) 19/18
(C) 20/19
(D) 21/20

20. A plano convex lens of refractive index μ_1 and focal length f_1 is kept in contact with another plano concave lens of refractive index μ_2 and focal length f_2. If the radius of curvature of their spherical faces is R each and $f_1 = 2f_2$, then μ_1 and μ_2 are related as:
(A) $\mu_1 + \mu_2 = 3$
(B) $2\mu_1 + \mu_2 = 1$
(C) $3\mu_2 + \mu_1 = 1$
(D) $2\mu_2 + \mu_1 = 1$

21. To get output '1' at R, for the given logic gate circuit the input values must be:
(A) X = 0, Y = 1
(B) X = 1, Y = 1
(C) X = 1, Y = 0
(D) X = 0, Y = 0

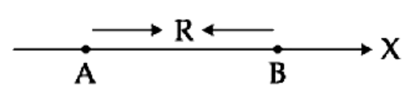


22. If the magnetic field of a plane electromagnetic wave is given by (The speed of light = 3×10^8 m/s) $B = 100 \times 10^{-6} \sin\left[2\pi \times 2 \times 10^{15}\left(t - \frac{x}{c}\right)\right]$ then the maximum electric field associated with it is:
(A) 6×10^4 N/C
(B) 3×10^4 N/C
(C) 4×10^4 N/C
(D) 4.5×10^4 N/C

23. Using a nuclear counter the count rate of emitted particles from a radioactive source is measured. At t = 0 it was 1600 counts per second and t = 8 seconds it was 100 counts per second. The count rate observed, as counts per second, at t = 6 seconds is close to:
(A) 200
(B) 150
(C) 400
(D) 360

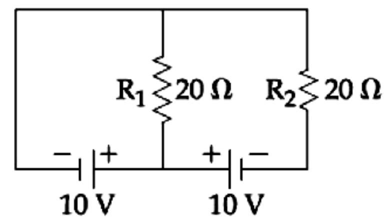
24. A solid metal cube of edge length 2 cm is moving in a positive y-direction at a constant speed of 6 m/s. There is a uniform magnetic field of 0.1 T in the positive z-direction. The potential difference between the two faces of the cube perpendicular to the x-axis, is:
(A) 12 mV
(B) 6 mV
(C) 1 mV
(D) 2 mV

25. Two electric dipoles A, B with respective dipole moments $\vec{d_A} = -4qa\hat{i}$ and $\vec{d_B} = 2qa\hat{i}$ are placed on the x-axis with a separation R, as shown in the figure. The distance from A at which both of them produce the same potential is:



(A) $\frac{R}{\sqrt{2}+1}$
(B) $\frac{\sqrt{2}R}{\sqrt{2}+1}$
(C) $\frac{R}{\sqrt{2}-1}$
(D) $\frac{\sqrt{2}R}{\sqrt{2}-1}$

26. In the given circuit the cells have zero internal resistance. The currents (in Amperes) passing through resistance R_1 and R_2 respectively, are:



(A) 1, 2
(B) 2, 2
(C) 0.5, 0
(D) 0, 1

27. A 2 W carbon resistor is color coded with green, black, red and brown respectively. The maximum current which can be passed through this resistor is:
(A) 20 mA
(B) 100 mA
(C) 0.4 mA
(D) 63 mA

28. A piece of wood of mass 0.03 kg is dropped from the top of a 100 m height building. At the same time, a bullet of mass 0.02 kg is fired vertically upward, with a velocity 100 ms^{-1}, from the ground. The bullet gets embedded in the wood. Then the maximum height to which the combined system reaches above the top of the building before falling below is: ($g = 10\ ms^{-2}$)
(A) 20 m
(B) 30 m
(C) 40 m
(D) 10 m

29. An insulating thin rod of length ℓ has a linear charge density $\rho(x) = \rho_0 \frac{x}{\ell}$ on it. The rod is rotated about an axis passing through the origin (x = 0) and perpendicular to the rod. If the rod makes n rotations per second, then the time averaged magnetic moment of the rod is:
(A) $\pi n \rho \ell^3$
(B) $\frac{\pi}{3} n \rho \ell^3$
(C) $\frac{\pi}{4} n \rho \ell^3$
(D) $n \rho \ell^3$

30. A homogeneous solid cylindrical roller of radius R and mass M is pulled on a cricket pitch by a horizontal force. Assuming rolling without slipping, angular acceleration of the cylinder is:
(A) $\frac{3F}{2mR}$
(B) $\frac{F}{3mR}$
(C) $\frac{F}{2mR}$
(D) $\frac{2F}{3mR}$

PART –B (CHEMISTRY)

31. The total number of isomers for a square planar complex $[M(F)(Cl)(SCN)(NO_2)]$ is
(A) 16 (B) 8
(C) 4 (D) 12

32. A process that has ΔH = 200 J mol^{-1} and ΔS = 40 JK^{-1} mol^{-1}. Out of the values given below, choose the minimum temperature above which the process will be spontaneous:
(A) 20 K (B) 12 K
(C) 5 K (D) 4 K

33. The values of K_p/K_c for the following reactions at 300 K are respectively:
(At 300 K, Rt = 24.62 dm^3 atm mol^{-1})

$$N_2(g) + O_2(g) = 2NO(g)$$
$$N_2O_4(g) = 2NO_2(g)$$
$$N_2(g) + 3H_2(g) = 2NH_3(g)$$

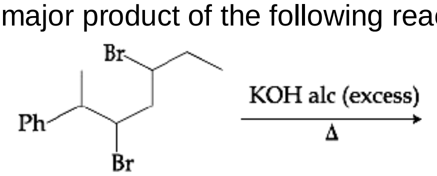
(A) 1, 24.62 dm^3 atm mol^{-1}, 606.0 dm^6 atm^2 mol^{-2}
(B) 1, 24.62 dm^3 atm mol^{-1}, 1.65×10^{-3} dm^{-6} atm^{-2} mol^2
(C) 1, 4.1×10^{-2} dm^{-3} atm^{-1} mol, 606.0 dm^6 atm^2 mol^{-2}
(D) 24.62 dm^3 atm mol^{-1}, 606.0 dm^6 atm^2 atm^2 mol^{-2}, 1.65×10^{-3} dm^{-6} atm^{-2} mol^2

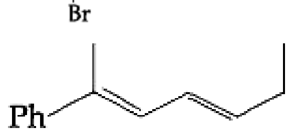
34. The total number of isotopes of hydrogen and number of radioactive isotopes among them, respectively are
(A) 3 and 1 (B) 3 and 2
(C) 2 and 1 (D) 2 and 0

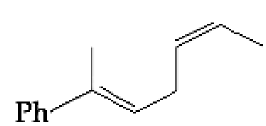
35. Water filled in two glasses A and B have BOD values of 10 and 20 respectively. The correct statement regarding them is
(A) B is more polluted than A
(B) A is suitable for drinking, whereas B is not
(C) both A and B are suitable for drinking
(D) A is more polluted than B

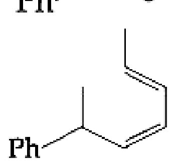
36. Which primitive unit cell has unequal edge length ($a \neq b \neq c$) and all axial angles different from 90°?
(A) Triclinic (B) Hexagonal
(C) Monoclinic (D) Tetragonal

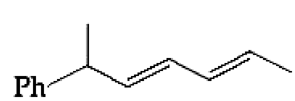
37. The major product of the following reaction is:



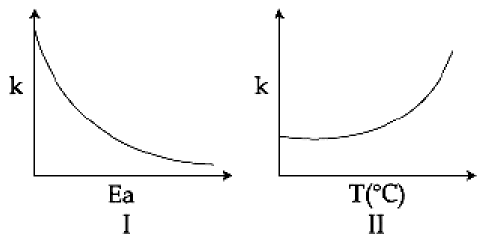
(A) 

(B) 

(C) 

(D) 

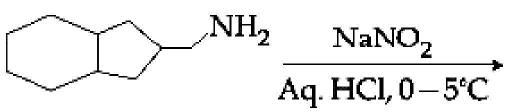
38. Consider the given plots for a reaction obeying Arrhenius equation (0°C < T < 300°C) : (k and E_a are rate constant and activation energy respectively)

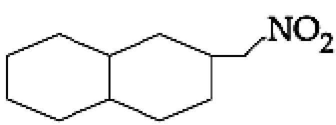


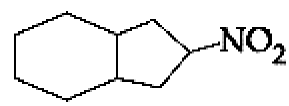
Choose the correct option:

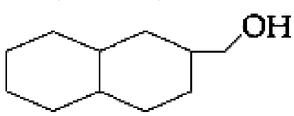
(A) I is right but II is wrong
(B) Both I and Ii are correct
(C) I is wrong but II is right
(D) Both I and II are wrong

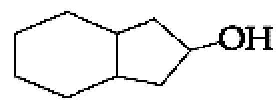
39. The major product formed in the reaction given below will be:



(A) 

(B) 

(C) 

(D) 

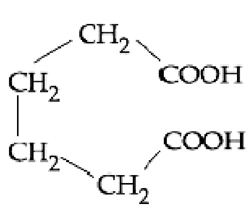
40. Wilkinson catalyst is

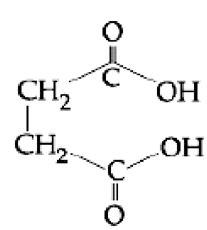
(A) $[(Ph_3P)_3IrCl]$
(B) $[Et_3P)_3RhCl]$
(C) $[(Ph_3P)_3RhCl]$
(D) $[(Et_3P)_3IrCl]$

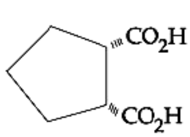
41. If dichloromethane(DCM) and water(H_2O) are used for differential extraction, which one of the following statements is correct?

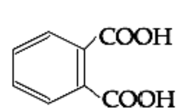
(A) DCM and H_2O would stay as lower and upper layer respectively in the S.F
(B) DCM and H_2O will make turbid/colloidal mixture
(C) DCM and H_2O would stay as upper and lower layer respectively in the separating funnel(S.F)
(D) DCM and H_2O will be miscible clearly

42. Which diccarboxylic acid in presence of a dehydrating agent is least reactive to give anhydride?

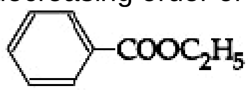
(A) 

(B) 

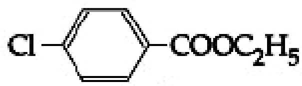
(C) 

(D) 

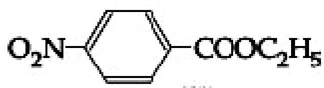
43. The decreasing order of ease of alkaline hydrolysis for the following ester is



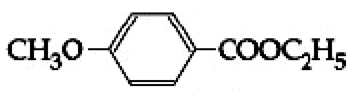
I



II



III

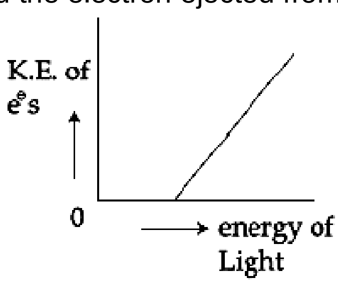


IV

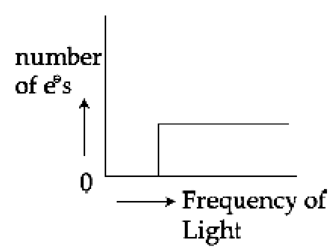
(A) III > II > IV > I (B) III > II > I > IV
(C) IV > II > III > I (D) II > III > I > IV

44. Which of the graphs shown below does not represent the relationship between incident light and the electron ejected from metal surface?

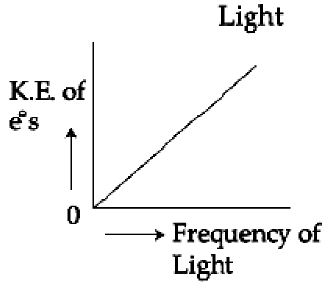
(A)



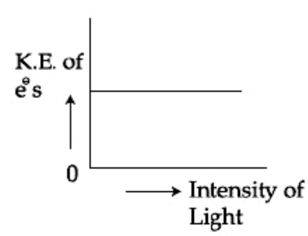
(B)



(C)



(D)



45. Which of the following is not an example of heterogeneous catalytic reaction?
(A) Ostwald's process (B) Combustion of coal
(C) Hydrogenation of vegetable oils (D) Haber's process

46. The effect of lanthanoid contraction in the lanthanoid series of elements by the large means
(A) increase in both atomic and ionic radii
(B) decrease in atomic radii and increase in ionic radii
(C) decrease in both atomic and ionic radii
(D) increase in atomic radii and decrease in ionic radii

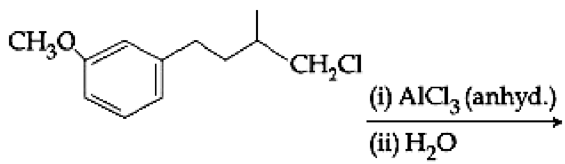
47. Which hydrogen in compound(E) is easily replaceable during bromination reaction in presence of light?

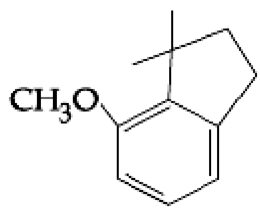
$$\underset{\delta}{CH_3}-\underset{\gamma}{CH_2}-\underset{\beta}{CH}=\underset{\alpha}{CH_2}$$

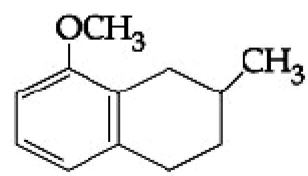
(E)

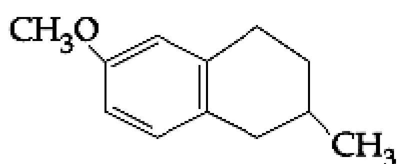
(A) α - hydrogen
(B) γ - hydrogen
(C) δ - hydrogen
(D) β - hydrogen

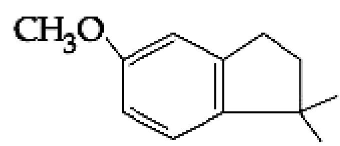
48. The major product of the following reaction is



(A) 

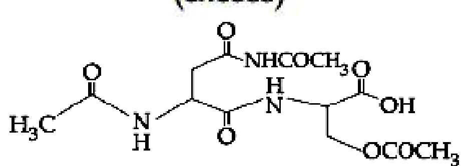
(B) 

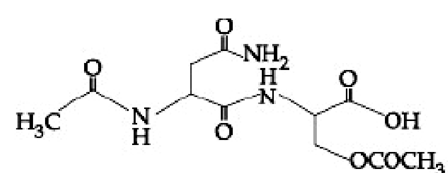
(C) 

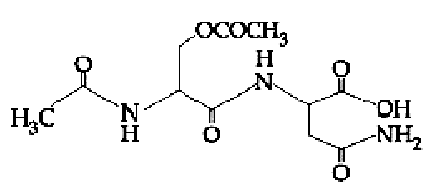
(D) 

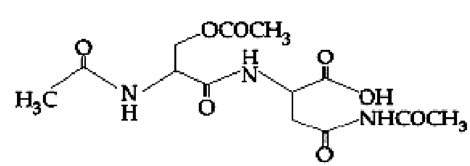
49. The correct structure of product ‘P’ in the following reaction is

$$\text{Asn}-\text{Ser} + \underset{\text{(excess)}}{(CH_3CO)_2O} \xrightarrow{NEt_3} P$$

(A) 

(B) 

(C) 

(D) 

50. The type of hybridisation and number of lone pair(s) of electrons of Xe in $XeOF_4$ respectively are
(A) sp^3d^2 and 1
(B) sp^3d and 2
(C) sp^3d^2 and 2
(D) sp^3d and 1

51. The electronegativity of aluminium is similar to
(A) carbon
(B) beryllium
(C) boron
(D) lithium

52. Consider the following

$$Zn^{2+} + 2e^- \longrightarrow Zn(s); E^o = -0.76\,V$$

$$Ca^{2+} + 2e^- \longrightarrow Ca(s); E^o = -2.87\,V$$

$$Mg^{2+} + 2e^- \longrightarrow Mg(s); E^o = -2.36\,V$$

$$Ni^{2+} + 2e^- \longrightarrow Ni(s); E^o = -0.25\,V$$

The reducing power of the metals increases in the order

(A) Ca < Zn < Mg < Ni
(B) Ni < Zn < Mg < Ca
(C) Zn < Mg < Ni < Ca
(D) Ca < Mg < Zn < Ni

53. The chemical nature of hydrogen peroxide is

(A) oxidising agent in acidic medium, but not in basic medium
(B) reducing agent in basic medium but not in acidic medium
(C) oxidising and reducing agent in acidic medium but not in basic medium
(D) oxidising and reducing agent in both acidic and basic medium

54. A mixture of 100 m mol of $Ca(OH)_2$ and 2 g of sodium sulphate was dissolved in water and the volume was made upto 100 mL. The mass of calcium sulphate formed and the concentration of OH^- in resulting solution, respectively are

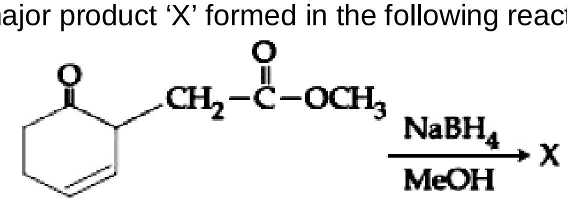
(Molar mass of $Ca(OH)_2$, Na_2SO_4 and $CaSO_4$ are 74, 143 and 136 g mol^{-1} respectively; K_{sp} of $Ca(OH)_2$ is 5.5×10^{-6})

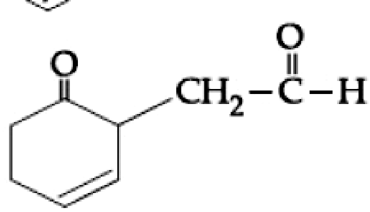
(A) 1.9 g, 0.28 mol L^{-1}
(B) 13.6 g, 0.28 mol L^{-1}
(C) 1.9g, 0.14 mol L^{-1}
(D) 13.6 g, 0.14 mol L^{-1}

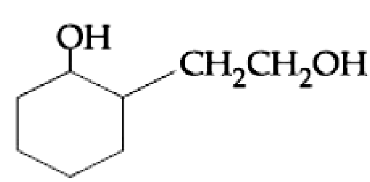
55. Liquids A and B form and ideal solution in the entire composition range. At 350 K, the vapour pressures of pure A and pure B are 7×10^3 Pa and 12×10^3 Pa, respectively. The composition of the vapour in equilibrium with a solution containing 40 mole percent of A at this temperature is

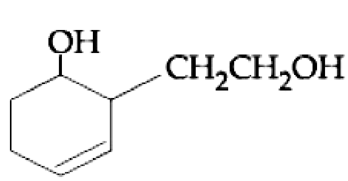
(A) $x_A = 0.37; x_B = 0.63$
(B) $x_A = 0.28; x_B = 0.72$
(C) $x_A = 0.4; x_B = 0.6$
(D) $x_A = 0.76; x_B = 0.24$

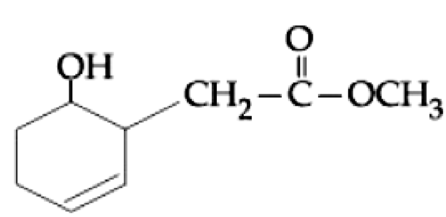
56. The major product 'X' formed in the following reaction is:



(A) 

(B) 

(C) 

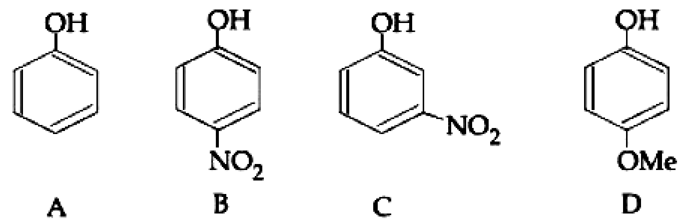
(D) 

57. The metal used for making X-ray tube window is

(A) Mg (B) Na

(C) Be (D) Ca

58. The increasing order of pKa values of the following compounds is



(A) C < B < A < D (B) B < C < D < A

(C) D < A < C < B (D) B < C < A< D

59. Hall-Herolut's process is given by

(A) $Cu^{2+}(aq) + H_2(g) \longrightarrow Cu(s) + 2H^+(aq)$

(B) $Cr_2O_3 + 2Al \longrightarrow Al_2O_3 + 2Cr$

(C) $2Al_2O_3 + 3C \longrightarrow 4Al + 3CO_2$

(D) $ZnO + C \xrightarrow{\text{Coke, 1673 K}} Zn + CO$

60. Two pi and half sigma bonds are present in

(A) O_2^+ (B) N_2

(C) O_2 (D) N_2^+

PART–C (MATHEMATICS)

61. An unbiased coin is tossed. If the outcome is a head then a pair of unbiased dice is rolled and the sum of the numbers obtained on the is noted. If the toss of the coin results in tail then a card from a well-shuffled pack of nine cards numbered 1, 2, 3,….., 9 is randomly picked and the number on the card is noted. The probability that the noted number is either 7 or 8 is:

(A) $\frac{13}{36}$ (B) $\frac{15}{72}$
(C) $\frac{19}{72}$ (D) $\frac{19}{36}$

62. The shortest distance between the point $\left(\frac{3}{2},0\right)$ and the curve $y=\sqrt{x},(x>0)$, is:

(A) $\frac{\sqrt{5}}{2}$ (B) $\frac{\sqrt{3}}{2}$
(C) $\frac{3}{2}$ (D) $\frac{5}{4}$

63. The plane passing through the point (4, –1, 2) and parallel to the lines $\frac{x+2}{3}=\frac{y-2}{-1}=\frac{z+1}{2}$ and $\frac{x-2}{1}=\frac{y-3}{2}=\frac{z-4}{3}$ also passes through the point:

(A) (1, 1< –1) (B) (1, 1, 1)
(C) (–1, –1, –1) (D) (–1, –1, 1)

64. The mean of five observations is 5 and their variance is 9.20. If three of the given five observations are 1, 3 and 8, then a ratio of other two observations is:

(A) 10 : 3 (B) 4 : 9
(C) 5 : 8 (D) 6 : 7

65. If 5, 5r, $5r^2$ are the lengths of the sides of a triangle, then r cannot be equal to:

(A) $\frac{3}{4}$ (B) $\frac{5}{4}$
(C) $\frac{7}{4}$ (D) $\frac{3}{2}$

66. If $\sum_{i=1}^{20}\left(\frac{{}^{20}C_{i-1}}{{}^{20}C_i+{}^{20}C_{i-1}}\right)^3=\frac{k}{21}$, then k equals:

(A) 400 (B) 50
(C) 200 (D) 100

67. The sum of all values of $\theta\in\left(0,\frac{\pi}{2}\right)$ satisfying $\sin^2 2\theta+\cos^4 2\theta=\frac{3}{4}$ is:

(A) π (B) $\frac{5\pi}{4}$
(C) $\frac{\pi}{2}$ (D) $\frac{3\pi}{8}$

68. Consider the quadratic equation $(c-5)x^2-2cs+(c-4)=0, c\neq 5.$ Let S be the set of all integral values of c for which one root of the equation lies in the interval (0, 2) and its other root lies in the interval (2, 3). Then the number of elements in S is:

(A) 18 (B) 12
(C) 10 (D) 11

69. If $\frac{dy}{dx}+\frac{3}{\cos^2 x}y=\frac{1}{\cos^2 x}, x\in\left(\frac{-\pi}{3},\frac{\pi}{3}\right)$ and $y\left(\frac{\pi}{4}\right)=\frac{4}{3}$, then $y\left(-\frac{\pi}{4}\right)$ equals:

(A) $\frac{1}{3}+e^6$ (B) $\frac{1}{3}$
(C) $-\frac{4}{3}$ (D) $\frac{1}{3}+e^3$

70. In a class of 140 students numbered 1 to 140, all even numbered students opted Mathematics course, those whose number is divisible by 3 opted Physics course and those whose number is divisible by 5 opted Chemistry course. Then the number of students who did not opt for any of the three courses is:

(A) 102 (B) 42
(C) 1 (D) 38

71. If the third term in the binomial expansion of $\left(1+x^{\log_2 x}\right)^5$ equals 2560, then a possible value of x is:

(A) $\frac{1}{4}$ (B) $4\sqrt{2}$
(C) $\frac{1}{8}$ (D) $2\sqrt{2}$

72. If the parabolas $y^2 = 4b(x-c)$ and $y^2 = 8ax$ have a common normal, then which one of the following is a valid choice for the ordered triad (a, b, c)?

(A) $\left(\frac{1}{2},2,3\right)$ (B) (1, 1, 3)
(C) $\left(\frac{1}{2},2,0\right)$ (D) (1, 1, 0)

73. If the system of equations

$x+y+z=5$

$x+2y+3z=9$

$x+3y+\alpha z=\beta$

has infinitely many solutions, then $\beta-\alpha$ equals:

(A) 21 (B) 8
(C) 18 (D) 5

74. For each t $\in$ R, let [t] be the greatest integer less than or equal to t. Then,

$$\lim_{x\to 1+}\frac{\left(1-|x|+\sin|1-x|\right)\sin\left(\frac{\pi}{2}[1-x]\right)}{|1-x|\,|1-x|}$$

(A) equals 1
(B) equals 0
(C) equals –1
(D) does not exist

75. Let d $\in$ R, and $A=\begin{bmatrix}-2 & 4+d & (\sin\theta)-2\\ 1 & (\sin\theta)+2 & d\\ 5 & (2\sin\theta)-d & (-\sin\theta)+2+2d\end{bmatrix}$, $\theta \in [0, 2\pi]$. If the minimum value of det(A) is 8, then a value of d is:

(A) –5
(B) –7
(C) $2\left(\sqrt{2}+1\right)$
(D) $2\left(\sqrt{2}+2\right)$

76. Let z_1 and z_2 be any two non-zero complex numbers such that $3|z_1|=4|z_2|$. If $z=\frac{3z_1}{2z_2}+\frac{2z_2}{3z_1}$ then:

(A) Re(z) = 0
(B) $|z|=\sqrt{\frac{5}{2}}$
(C) $|z|=\frac{1}{2}\sqrt{\frac{17}{2}}$
(D) Im(z) = 0

77. Let $I=\int_a^b\left(x^4-2x^2\right)dx$. If I is minimum then the ordered pair (a, b) is:

(A) $\left(0,\sqrt{2}\right)$
(B) $\left(-\sqrt{2},0\right)$
(C) $\left(\sqrt{2},-\sqrt{2}\right)$
(D) $\left(-\sqrt{2},\sqrt{2}\right)$

78. A point P moves on the line 2x – 3y + 4 = 0. If Q(1, 4) and R(3, –2) are fixed points, then the locus of the centroid of ΔPQR is a line:

(A) with slope $\frac{3}{2}$
(B) parallel to x-axis
(C) with slope $\frac{2}{3}$
(D) parallel to y-axis

79. Let $f(x)=\begin{cases}\max\{|x|,x^2\}, & |x|\le 2\\ 8-2|x|, & 2<|x|\le 4\end{cases}$. Let S be the set of points in the interval (–4, 4) at which f is not differentiable. Then S:

(A) is an empty set
(B) equals {–2, –1, 0, 1, 2}
(C) equals {–2, –1, 1, 2}
(D) equals {–2, 2}

80. If a circle C passing through the point (4, 0) touches the circle $x^2 + y^2 + 4x - 6y = 12$ externally at the point (1, –1), then the radius of C is:

(A) $2\sqrt{5}$
(B) 4
(C) 5
(D) $\sqrt{57}$

81. Let f : R → R be a function such that $f(x) = x^3 + x^2 f'(1) + x f''(2) + f'''(3)$, $x \in R$. Then f(2) equals:

(A) –4
(B) 30
(C) –2
(D) 8

82. Let $\vec{a} = 2\hat{i} + \lambda_1\hat{j} + 3\hat{k}, \vec{b} = 4\hat{i} + (3 - \lambda_2)\hat{j} + 6\hat{k}$ and $\vec{c} = 3\hat{i} + 6\hat{j} + (\lambda_3 - 1)\hat{k}$ be three vectors such that $\vec{b} = 2\vec{a}$ and $\vec{a}$ is perpendicular to $\vec{c}$. Then a possible value of $(\lambda_1, \lambda_2, \lambda_3)$ is:

(A) (1, 3, 1)
(B) $\left(-\frac{1}{2}, 4, 0\right)$
(C) $\left(\frac{1}{2}, 4, -2\right)$
(D) (1, 5, 1)

83. Let A be a point on the line $\vec{r} = (1 - 3\mu)\hat{i} + (\mu - 1)\hat{j} + (2 + 5\mu)\hat{k}$ and B(3, 2, 6) be a point in the space. Then the value of μ for which the vector $\overrightarrow{AB}$ is parallel to the plane x – 4y + 3z = 1 is:

(A) $\frac{1}{4}$
(B) $\frac{1}{8}$
(C) $\frac{1}{2}$
(D) $-\frac{1}{4}$

84. Let $n \geq 2$ be a natural number and $0 < \theta < \pi/2$. Then $\int \frac{(\sin^n\theta - \sin\theta)^{\frac{1}{n}}\cos\theta}{\sin^{n+1}\theta} d\theta$ is equal to: (where C is a constant of integration)

(A) $\frac{n}{n^2 - 1}\left(1 - \frac{1}{\sin^{n-1}\theta}\right)^{\frac{n+1}{n}} + C$
(B) $\frac{n}{n^2 + 1}\left(1 - \frac{1}{\sin^{n-1}\theta}\right)^{\frac{n+1}{n}} + C$
(C) $\frac{n}{n^2 - 1}\left(1 + \frac{1}{\sin^{n-1}\theta}\right)^{\frac{n+1}{n}} + C$
(D) $\frac{n}{n^2 - 1}\left(1 - \frac{1}{\sin^{n+1}\theta}\right)^{\frac{n+1}{n}} + C$

85. Consider a triangular plot ABC with sides AB = 7 m, BC = 5 m and CA = 6 m. A vertical lamp-post at the mid point D of AC subtends an angle 30° at B. The height (in m) of the lamp-post is:

(A) $\frac{3}{2}\sqrt{21}$
(B) $\frac{2}{3}\sqrt{21}$
(C) $2\sqrt{21}$
(D) $7\sqrt{21}$

86. The equation of a tangent to the hyperbola $4x^2 - 5y^2 = 20$ parallel to the line x – y = 2 is:

(A) x – y + 1 = 0
(B) x – y + 7 = 0
(C) x – y + 9 = 0
(D) x – y – 3 = 0

87. If the line $3x + 3y - 24 = 0$ intersects the x-axis at the point A and the y-axis at the point B, then the incentre of the triangle OAB, where O is the origin, is:
(A) (3, 4)
(B) (2, 2)
(C) (4, 3)
(d) (4, 4)

88. The sum of all two digit positive numbers which when divided by 7 yield 2 or 5 as remainder is:
(A) 1256
(B) 1465
(C) 1365
(D) 1356

89. If the area enclosed between the curves $y = kx^2$ and $x = ky^2$, $(k > 0)$, is 1 square unit. Then k is:
(A) $\frac{\sqrt{3}}{2}$
(B) $\frac{1}{\sqrt{3}}$
(C) $\sqrt{3}$
(D) $\frac{2}{\sqrt{3}}$

90. Consider the statement: "$P(n): n^2 - n + 41$ is prime." Then which one of the following is true?
(A) Both P(3) and P(5) are true
(B) P(3) is false but P(5) is true
(C) Both P(3) and P(5) are false
(D) P(5) is false but P(3) is true

HINTS AND SOLUTIONS

PART A – PHYSICS

1. $k = \frac{P^2}{2m} = \frac{h^2}{2m\lambda^2}$

$$= \frac{(6.62\times10^{-34})^2}{2\times9.1\times10^{-31}\times(7.5\times10^{-12})^2}\times\frac{1}{1.6\times10^{-19}}\,eV$$

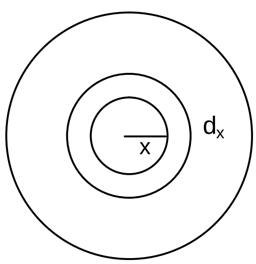
Alternate method

$\lambda = \sqrt{\frac{150}{V}}$

$\Rightarrow \quad 7.5 \times 10^{-12} = \sqrt{\frac{150}{V}}$

$V = \frac{80}{3}kV$

Energy = $\frac{80}{3}keV \simeq 25\,keV$

2. 

$d\tau = (\mu\, dN)x$

$= \mu\left(\frac{F}{\pi R^2}\times 2\pi\times dx\right)x$

$\tau = \int_0^R d\tau$

3. $V_{AJ} = IR_{AJ}$

$\frac{E}{3} = \left(\frac{E}{12r+3r}\right)\times\left(\frac{x}{L}\times 12r\right)$

4. $D = \sqrt{2\,h_T R} + \sqrt{2h_R R}$

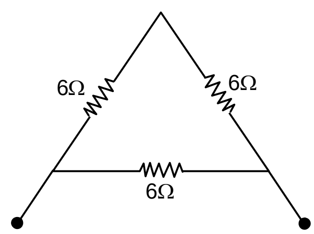
5. $1\left(0, \frac{a}{2}, \frac{a}{2}\right)$

$2\left(\frac{a}{2}, \frac{a}{2}, 0\right)$

$\vec{r}_2 - \vec{r}_1 = \frac{a}{2}\hat{i} - \frac{a}{2}\hat{k}$

Unit vector = $\frac{\hat{i}-\hat{k}}{\sqrt{2}}$

6.



$R_{net} = 4\Omega$

7. $W = F\, s \cos\theta$

$$= m\left(g+\frac{g}{2}\right)\times\left[\frac{1}{2}\times\frac{g}{2}\times t^2\right]$$

8. For maxima,

$d\sin\theta = n\lambda$

$\lambda = \dfrac{d\theta}{n}$ $\quad$ ($\because$ $\sin\theta \approx \theta$ when θ is small)

$= \dfrac{2500}{n}$ nm

Take n = 4, 5 for λ = 625 nm and 500 nm.

9. Area $\propto \pi(\text{Range})^2 \propto v^4$

$\therefore \quad \dfrac{A_1}{A_2} = \left(\dfrac{1}{2}\right)^4$

10. $\dfrac{128\,\text{kg}}{m^3} = \dfrac{64\,(2\text{kg})}{10\,\text{cm}^3}$

$$= \frac{64\,(2\,\text{kg})}{10^6\,(50\,\text{cm})^3}\times 50^3$$

$$= \frac{64\times50\times50\times50}{100\times100\times100}\times\frac{2\,\text{kg}}{(50\,\text{cm})^3} = 8\frac{(2\,\text{kg})}{(50\,\text{cm}^3)}$$

11. Work done by external agent = $U_f - U_i$

= 2 MB

12. $\dfrac{\Delta Q}{\Delta t} = \dfrac{kA}{\ell}(T_2 - T_1)$

$$\frac{1}{A}\left(\frac{\Delta Q}{\Delta t}\right) = \frac{k}{\ell}(T_2 - T_1)$$

13. $C_{net} = C_1 + C_2 + C_3$

$$\frac{kAE_o}{d} = k_1\left(\frac{A}{3}\right)\frac{E_o}{d} + k_2\left(\frac{A}{3}\right)\frac{E_o}{d} + k_3\left(\frac{A}{3}\right)\frac{E_o}{d}$$

$$k = \frac{k_1+k_2+k_3}{3} = 12$$

14. $Q_1 + q_2 + Q_3 = Q$ …(1)

$$\frac{Q_1}{4\pi a^2} = \frac{Q_2}{4\pi b^2} = \frac{Q_3}{4\pi c^2} = k \quad \text{…(2)}$$

Subs. Q_1, Q_2, Q_3 in (1)

$$k = \frac{Q}{4\pi(a^2 + b^2 + c^2)}$$

$$v = \frac{kQ_1}{a} + \frac{kQ_2}{b} + \frac{kQ_3}{c}$$

15. $n_1 = n_2 = n_3$

$$\Rightarrow \quad 1 - \frac{T_2}{T_1} = 1 - \frac{T_3}{T_2} = 1 - \frac{T_4}{T_3}$$

$$\Rightarrow \quad \frac{T_2}{T_1} = \frac{T_3}{T_2} = \frac{T_4}{T_3}$$

$$\Rightarrow \quad T_2T_3 = T_1T_4 \text{ and } \frac{T_3^2}{T_2} = T_4$$

Solve for T_2 and T_3.

16. Initially, kinetic energy = $\frac{1}{2}mv^2 = \frac{1}{2}m\frac{GM_e}{r}$

By conservation of M.E.,

$$\frac{-GM_em}{r} + KE = 0$$

$$KE = \frac{GM_em}{r} = mv^2$$

17. $\frac{dV}{dt} = \phi - a\sqrt{2gh} = 0$ (for maximum height)

$$h = \frac{\phi^2}{2ga^2} = \frac{10^{-8}}{2 \times 9.8 \times 10^{-8}} = 5.1 \text{ cm}$$

18. $f = \frac{n}{2\ell}\sqrt{\frac{T}{m}}$

On solving, n = 5,
5 loops are formed in 1 m.

$\therefore$ Separation between successive nodes = $\frac{1}{5}$m = 20 cm

19. $f_1 = f_o\left(\frac{340}{340 - 17}\right)$; $f_2 = f_o\left(\frac{340}{340 - 34}\right)$

$$\frac{f_1}{f_2} = \frac{306}{323} \text{ or } \frac{18 \times 17}{19 \times 17} \text{ or } \frac{18}{19}$$

20. $\dfrac{1}{f_1} = (\mu_1 - 1)\left(\dfrac{1}{R} - \dfrac{1}{\infty}\right)$; $\dfrac{1}{f_2} = (\mu_2 - 1)\left(\dfrac{1}{\infty} - \dfrac{1}{R}\right)$

$\dfrac{1}{f_1} = \dfrac{(\mu - 1)}{R}$; $\dfrac{1}{f_2} = -\left(\dfrac{\mu_2 - 1}{R}\right)$

$f_2 = 2f_1$; $\dfrac{1}{f_1} = \dfrac{2}{f_2}$

$\dfrac{\mu_1 - 1}{R} = \dfrac{-2(\mu_2 - 1)}{R}$

$\mu_1 + 2\mu_2 = 3$

21. $R = \overline{P + Q} = \overline{(\bar{x} + y) + (\overline{x\bar{y}})}$

$= \left(\overline{\bar{x} + y}\right) \cdot \left(x\bar{y}\right)$

$= \left(x \cdot \bar{y}\right) \cdot \left(x\bar{y}\right)$

$= x\bar{y}$

22. E = CB

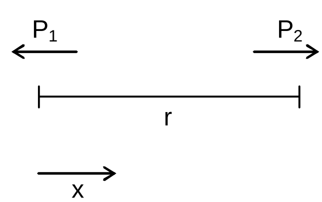
23. In 8 seconds count rate becomes $\dfrac{1}{16}$ times.

$\therefore$ 4 half lives = 8s
1 half lie = 2s

In 6s or 3 half lives, count rate = $\dfrac{1600}{2^3}$ = 200

24. E = vB
v = Ed = dvB

25.



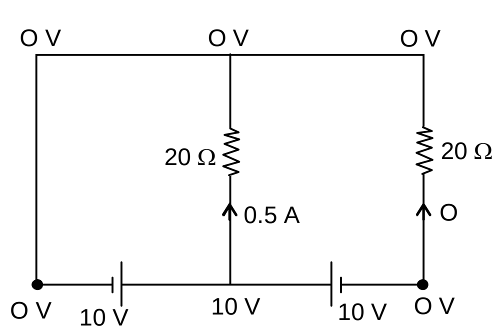
$v_1 = v_2$

$\dfrac{k(2qa)}{x^2} = \dfrac{k(4qa)}{(R - x)^2}$

$R - x = \sqrt{2}\,x$

$x = \dfrac{R}{\sqrt{2} + 1}$

26.



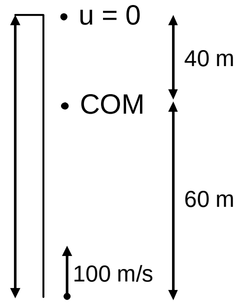
27. By colour coding,

$R = 50 \times 10^2\ \Omega$

$I^2R = P$

$I = \sqrt{\frac{P}{R}} = 20\ \text{mA}$

28.



Y_{cm} from ground $= \frac{0.03 \times 100}{0.05} = 60$ m

$V_{cm} = \frac{0.02 \times 100}{0.05} = 40$ m/s

$H = \frac{V_{cm}^2}{2g} = \frac{40 \times 40}{20} = 80$ m

Height above building = 80 – 40 = 40 m

29. $dM = di\, A$

$= \left(\frac{dq\omega}{2\pi}\right)\pi x^2$

$= (\rho dx)\frac{\omega}{2\pi}\pi x^2$

$M = \int_0^L dM$

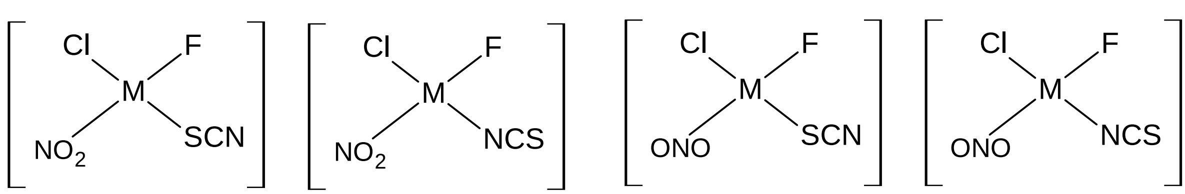
30. $F - f = ma$

$fR = \frac{1}{2}mR^2\alpha$

$a = \alpha R$

PART B – CHEMISTRY

31.



= 3 GI = 3 GI = 3 GI = 3 GI

Total = 12 GI

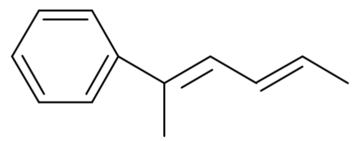
32. $\Delta S = \frac{\Delta H}{T}$

33. $K_P = K_C(RT)^{\Delta ng}$

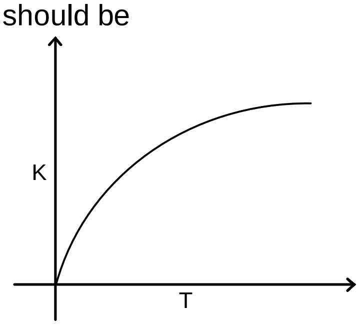
34. Fact based

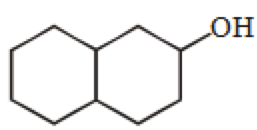
35. More polluted water has high biological oxygen demand.

36. Fact based.

37. Major product will be  Due to E_2 elimination & extended conjugation

38. $K = Ae^{-E_a/RT}$
(i) is true but (ii) is false
(ii) should be



39. Answer should be 

40. Fact based

41. Density of DCM is higher than H_2O.

42. Adipic acid will give unstable 7 membered anhydride.

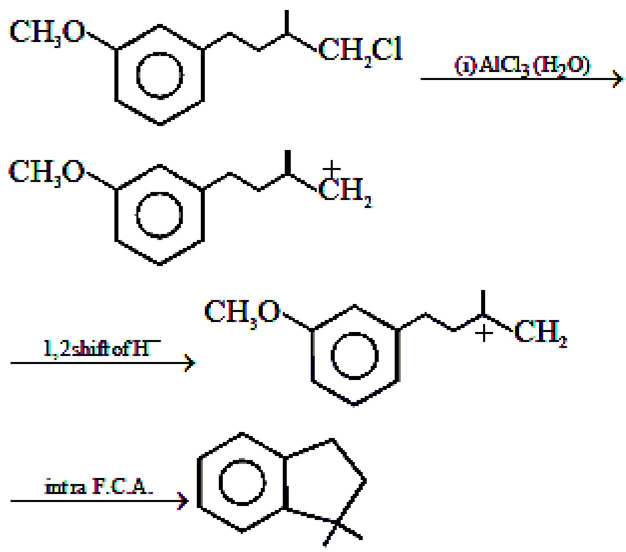
43. Electron withdrawing group enhances the rate of hydrolysis.

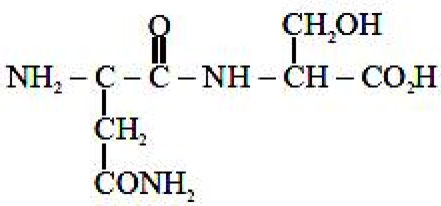
44. $K.E = h\nu - h\nu_0$, So (B) is not correct.

45. Fact based.

46. Atomic & ionic radii decreases

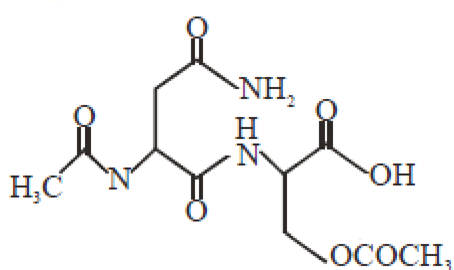
47. Allylic radical will form in presence of sunlight.

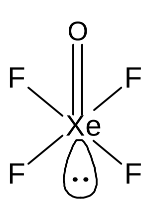
48. 

49. 

$$\text{Asn} - \text{Ser} + \underset{\text{excess}}{(CH_3CO)_2O} \xrightarrow{NEt_3} P$$

P is



50. 

51. Diagonal relationship.

52. Lesser the SRP value higher the reducing power.

53. Fact based.

54. $Ca(OH)_2 + Na_2SO_4 \longrightarrow CaSO_4 + 2NaOH$

100 m mol	14 m mol	—	—
—	—	14 m mol	28 m mol

$w_{CaSO_4} = 14\times10^{-3}\times136 = 1.9gm$

$[OH^-] = \frac{28}{100} = 0.28M$

55. $P_A = \chi_A P_A^0 = Y_A P_T$

$P_T = \chi_A P_A^0 + \chi_B P_B^0 = 0.4 \times 7 \times 10^3 + 0.6 \times 12 \times 10^3 = 10^4$

$0.4 \times 7 \times 10^3 = Y_A \times 10^4$

$Y_A = 0.28$

$Y_B = 1 - 0.28 = 0.72$

56. $NaBH_4$ don't reduce ester.

57. Fact based

58. Higher the Ka value lower the pKa value.

59. Hall-Herolut's process is given by

$2Al_2O_3 + 3C \longrightarrow 4Al + 3CO_2$

$2Al_2O_3(\ell) \rightleftharpoons 4Al^{3+}(\ell) + 6O^{2-}(\ell)$

At cathode: $4Al^{3+}(\ell) + 12e^- \longrightarrow 4Al(\ell)$

At anode: $6O^{2-}(\ell) \longrightarrow 3O_2(g) + 12e^-$

$2C + 3O_2 \longrightarrow 3CO_2(\uparrow)$

60

$N_2^{\oplus} \Rightarrow BO = 2.5 \Rightarrow \left[\pi - \text{Bond} = 2 \;\&\; \sigma - \text{Bond} = \frac{1}{2}\right]$

$N_2 \Rightarrow B.O. = 3.0 \Rightarrow [\pi\text{-Bond} = 2 \;\&\; \sigma\text{-Bond} = 1]$

$O_2^{\oplus} = B.O. \Rightarrow 2.5 \Rightarrow [\pi\text{-Bond} = 1.5 \;\&\; \sigma\text{-Bond} = 1]$

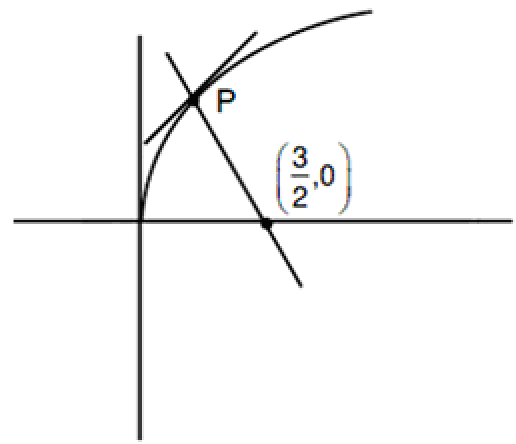
$O_2 \Rightarrow B.O. \Rightarrow 2 \Rightarrow [\pi\text{-Bond} \Rightarrow 1 \;\&\; \sigma\text{-Bond} = 1]$

PART C – MATHEMATICS

61. P (7 or 8)

$=P(H)P(7\text{ or }8)+P(T)P(7\text{ or }8)=\frac{1}{2}\times\frac{11}{36}+\frac{1}{2}\times\frac{2}{9}=\frac{11}{72}+\frac{1}{9}=\frac{19}{72}$

62. Let P be the point nearest to $\left(\frac{3}{2},0\right)$, then normal at P will pass through $\left(\frac{3}{2},0\right)$.



Let Co – ordinates of P be $s\left(\frac{t^2}{4},\frac{t}{2}\right)$

Hence equation of normal is $y+tx=\frac{t}{2}+\frac{t^2}{4}$

The line passes through $\left(\frac{3}{2},0\right)$

$\frac{3t}{2}=\frac{t}{2}+\frac{t^3}{4}\Rightarrow t=2$ (–2, 0 are rejected)

hence nearest point is (1, 1)

distance $\sqrt{\left(\frac{3}{2}-1\right)^2+(1-0)^2}=\frac{\sqrt{5}}{2}$

63. $\vec{n}=\vec{n}_1\times\vec{n}_2=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 3 & -1 & 2\\ 1 & 2 & 3\end{vmatrix}=7\hat{i}-7\hat{j}+7\hat{k}$

Equation of plane is $-7(x-4)-7(y+1)+7(z-2)=0$

$\Rightarrow -7x-7y+7z+7=0\Rightarrow x+y-z=1$

64. $\mu=\frac{1+3+8+x+y}{5}$

$25=12+x+y\Rightarrow x+y=13$ ....(1)

$\sigma^2=\frac{\sum(x_i-\mu)^2}{N}$

$9.2=\frac{1+9+64+x^2+y^2}{5}-25$

$34.2\times5=74+x^2+y^2$

$171=74+x^2+y^2$

$97=x^2+y^2$ ....(2)

$(x+y)^2=x^2+y^2+2xy$

$169-97=2xy\Rightarrow xy=36$

T = 4, 9

So ratio is $\frac{4}{9}$ or $\frac{9}{4}$

65. $5, 5r, 5r^2$ sides of triangle,

$5+5r>5r^2$ ………………(1)

$5+5r^2>5r$ ………………(2)

$5r+5r^2>5$ ………………(3)

From $r^2-r-1<0$

$\left[r-\left(\frac{1-\sqrt{5}}{2},\frac{1+\sqrt{5}}{2}\right)\right]$ ………..(4)

From (2)

$r^2-r+1>0 \Rightarrow r \in R$ ……….(5)

From (3)

$r^2+r-1>0$

So, $\left(r+\frac{\sqrt{1+\sqrt{5}}}{2}\right)\left(r+\frac{1-\sqrt{5}}{2}\right)>0$

$r \in \left(-\infty, \frac{1+\sqrt{5}}{2}\right) \cup \left(-\frac{1-\sqrt{5}}{2}, \infty\right)$ …………(6)

From (4), (5), (6) $r \in \left(\frac{-1+\sqrt{5}}{2}, \frac{1+\sqrt{5}}{2}\right)$

now check options

66. $\sum_{i=1}^{20}\left(\frac{{}^{20}C_{I-1}}{{}^{20}C_I+{}^{20}C_{I-1}}\right)^3$

Now $\frac{{}^{20}C_{I-1}}{{}^{20}C_I+{}^{20}C_{I-1}}=\frac{{}^{20}C_{I-1}}{{}^{21}C_I}=\frac{I}{21}$

Let given sum be S, so

$S=\sum_{I=1}^{20}\frac{(i)^3}{21^3}=\frac{1}{(21)^3}\left(\frac{20.21}{2}\right)^2=\frac{100}{21}$

Given $S=\frac{k}{21} \Rightarrow k=100$

67. $\sin^2 2\theta+\cos^4 2\theta=\frac{3}{4}$

Let $\cos^2 2\theta=t$

$\Rightarrow 1-\cos^2 2\theta+\cos^4 2\theta=\frac{3}{4}$

$\Rightarrow t=\frac{1}{2} \Rightarrow \cos^2 2\theta=\frac{1}{2}$

$\Rightarrow 2\cos^2 2\theta - 1 = 0 \Rightarrow \cos 4\theta = 0$

$\Rightarrow 4\theta = (2n+1)\frac{\pi}{2}$

$\Rightarrow \theta = (2n+1)\frac{\pi}{8} \Rightarrow \theta = \frac{\pi}{8}, \frac{3\pi}{8} \in \left[0, \frac{\pi}{2}\right]$

Sum of values of θ is $\frac{\pi}{2}$

68. Case – 1

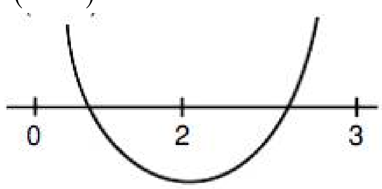
$c - 5 > 0$ ....(i)

$f(0) > 0$

$c - 4 > 0$ ....(ii)

$f(2) < 0$

$4(c-5) - 4c + c - 4 < 0$



$c < 24$ ....(iii)

$f(2) > 0$

$9(c-5) - 6c + c - 4 > 0$

$4c - 49 > 0 \Rightarrow c > \frac{49}{4}$ ....(iv)

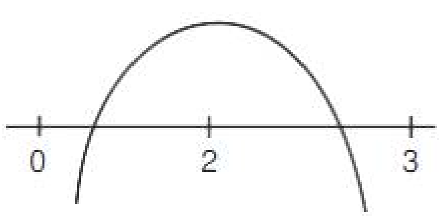
Here $(i) \cap (ii) \cap (iii) \cap (iv)$

$c \in \left(\frac{49}{4}, 24\right)$

Case – II

$c - 5 < 0$ ....(i)

$f(0) < 0$



$c < 4$ ....(ii)

$f(2) > 0 \Rightarrow c > 24$ ....(iii)

$f(3) < 0 \Rightarrow c < 49$ ....(iv)

$4 \Rightarrow c \in \phi$

$c \in \left(\frac{49}{4}, 24\right)$

69. $\frac{dy}{dx}+\left(3\sec^2 x\right)y=\sec^2 x$

This is linear differential equation

Integrating factor $=e^{\int 3\sec^2 x dx}=e^{3\tan x}$

Hence $y.e^{3\tan x}=e^{\int 3\tan x}.\sec^2 x\,dx$

$\Rightarrow y e^{3\tan x}=\frac{e^{3\tan x}}{3}+c$

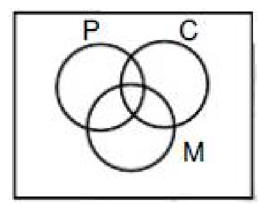
$\Rightarrow y=Ce^{-3\tan x}+\frac{1}{3}$

Given $y\left(\frac{\pi}{4}\right)=\frac{4}{3}\Rightarrow\frac{4}{3}=Ce^{-3}+\frac{1}{3}$

$\Rightarrow C=e^3$

Hence $y\left(\frac{\pi}{4}\right)=e^3.e^3+\frac{1}{3}=e^6+\frac{1}{3}$

70. $n(p)=\left[\frac{140}{3}\right]=46$



$n(C)=\left[\frac{140}{5}\right]=28$

$n(M)=\left[\frac{140}{2}\right]=70$

$n(P\cup C\cup M)=n(P)+n(C)+n(M)-n(P\cap C)-n(C\cap M)-n(M\cap P)+n(P\cap M\cap C)$

$=46+28+70-\left[\frac{140}{15}\right]-\left[\frac{140}{10}\right]-\left[\frac{140}{6}\right]+\left[\frac{140}{30}\right]$

$=144-9-14-23+4=102$

So required number of student $=140-102=38$

71. In the expansion of $\left(1+x^{\log_2 x}\right)^5$

third term say $T_3={}^5C_2\left(x^{\log_2 x}\right)^2=2560$

$\Rightarrow\left(x^{lo_2 x}\right)^2=256$

taking lograthium to the base 2 on both sides

$\Rightarrow 2(\log_2 x)^2=8\Rightarrow(\log_2 x)=\pm 2$

$\Rightarrow x=4,\frac{1}{4}$

Here $x=\frac{1}{4}$

72. Normal to there 2 curves are

$y=m(x-c)-2bm-bm^3$

$y=mx-4am-2am^3$

If they how a common normal

(c + 2b) – 4a = (2a – b) m^2 (m = 0 corresponds to axis)

$$\Rightarrow m^2 = \frac{c}{2a-b} - 2 > 0$$

$$\frac{c}{2a-b} > 2$$

According to the question answer is (1, 2, 3, 4) but my be in question they want common normal other than x – axis, hence answer is (B)

73. $x + y - z = 5$

$x + 2y + 3z = 9,$

$x + 3y + \alpha z = \beta$

$$D = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 3 & 0 \end{vmatrix} = 0 \Rightarrow (2\alpha - 9) + (3 - \alpha) + (3 - 2) = 0 \Rightarrow \alpha = 5$$

$$\text{Now, } D_3 = \begin{vmatrix} 1 & 1 & 5 \\ 1 & 2 & 9 \\ 1 & 3 & \beta \end{vmatrix} = 0 \Rightarrow 2\beta - 27 + 9\beta - 5(3 - 2) = 0 \Rightarrow \beta = 13$$

$\Rightarrow$ at $\alpha = 5, b = 13$ above 3 planes from common line

74. $$\lim_{x \to 1+} \frac{\left(1 - |x| + \sin|1 - x|\right)\sin\left([1 - x]\frac{\pi}{2}\right)}{|1 - x|[1 - x]}$$

$$= \lim_{x \to 1^-} \frac{\left(1 - x + \sin(x - 1)\right)\sin\left(-\frac{\pi}{2}\right)}{(x - 1)(-1)}$$

$$= \lim_{x \to 1^-} \frac{-(x - 1) + \sin(x - 1)}{(x - 1)} = -1 + 1 = 0$$

75. $$|A| = \begin{vmatrix} -2 & 4 + d & (\sin\theta - 2) \\ 1 & (\sin\theta) + 2 & d \\ 5 & (2\sin\theta) - d & (-\sin\theta) + 2 + 2d \end{vmatrix}$$

$$= \begin{vmatrix} -2 & 4 + d & (\sin\theta - 2) \\ 1 & (\sin\theta) & d \\ 1 & 0 & 0 \end{vmatrix} \qquad (\text{New } R_3 = R_3 - 2R_2 + R_1)$$

$$= (4 + d)d - \sin^2\theta + 4 = (d + 2)^2 - \sin^2\theta$$

Because minimum value of $|A| = 8 \Rightarrow (d + 2)^2 = 9 \Rightarrow d = 1$ or -5

76. $\left|\frac{3z_1}{2z_2}\right| = 2$

Let $\frac{3z_1}{2z_2} = 2\cos\theta + 2(\sin\theta)i$

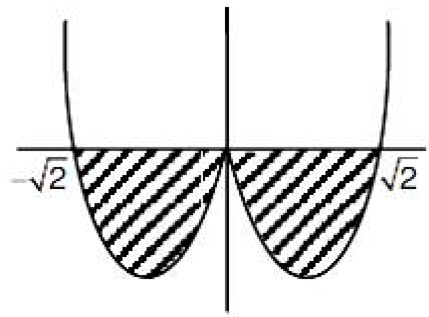
$\Rightarrow \frac{2z_2}{3z_1} = \frac{1}{2}\cos\theta - \frac{1}{2}(\sin\theta)$

Given, $z = \frac{2z_1}{3z_2} + \frac{3z_2}{2z_1} = \frac{5}{2}\cos\theta + \frac{3}{2}(\sin\theta)i$

Which is neither purely real nor purely imaginary and $|z|$ depends on θ.

77. $\int_a^b \left(x^4 - 2x^2\right)dx$

From figure min area is $\left(-\sqrt{2}, \sqrt{2}\right)$



78. Let point P is (α, β) and centroid of ΔPQR is (h, k), then $3h = \alpha + 1 + 3$ and $3k = \beta + 4 - 2$

$\Rightarrow \alpha = 3h - 4$ and $\beta = 3k - 2$

Because (α, β) lies on $2x - 3y + 4 = 0$

$\Rightarrow 2(3h - 4) - 3(k - 2) + 4 = 0$

$\Rightarrow$ locus is $6x - 9y + 2 = 0$ whose slope is $\frac{2}{3}$

79. From the graph we can easily conclude that $f(x)$ is non – derivable at $x = -2, -1, 0, 1, 2$

80. Tangent at (1, –1) is $x(1) + y(-1) + 2(x + 1) - 3(y - 1) - 12 = 0$

$= 3x - 4y = 7$

Required circle is $(x - 1)^2 + (y + 1)^2 + \lambda(3x - 4y - 7) = 0$

It pass through (4, 0)

$\Rightarrow 9 + 1 + \lambda(12 - 7) = 0 \Rightarrow \lambda = -2$

$\Rightarrow$ required circle is $x^2 + y^2 - 8x + 10y + 16 = 0$

$\Rightarrow$ Radius $= \sqrt{16 + 25 - 16} = 5$

81. $f(x) = x^3 + ax^2 + bx + c$

$f'(x) = 3x^2 + 2ax + b$

$f''(x) = 6x + 2a$

$f'''(x) = 6 \; a = f'(1) = 3 + 2a + b \quad \Rightarrow a + b = -3$

$b = f'' = 12 + 2a \quad \Rightarrow 2a - b = -12$

$c = f'''(3) \quad \Rightarrow c = 6$ and $a = -5, b = 2$

$\Rightarrow f(x) = x^2 - 5x^2 + 2x + 6$

$\Rightarrow f(2) = 8 - 20 + 4 + 6 = -2$

82. Because $b = 2\vec{a}$, so $3 - \lambda_2 = 2\lambda_1$ ....(i)

Because a is perpendicular to c so $6 + 6\lambda_1 + 3(\lambda_3 - 1) = 0$ ....(ii)

$\Rightarrow (\lambda_1, \lambda_2, \lambda_3) = (\lambda_1, 3 - 2\lambda_1, -1 - 2\lambda_1)$ where $\lambda_1 \in R$

$\Rightarrow \left(-\frac{1}{2}, 4, 0\right)$ satisfied above triplet.

83. Let A is $(1 - 3\mu, \mu - 1, 2 + 5\mu)$

$\overrightarrow{AB} = (3\mu + 2)i + (3 - \mu)j + (4 - 5\mu)$

$\hat{k}$ which is parallel to plane $x - 4y + 3z = 1$

$\Rightarrow 1(3\mu + 2) - 4(3 - \mu) + 3(4 - 5\mu) = 0$

$= -8\mu + 2 = 0 \Rightarrow \mu = \frac{1}{4}$

84. $\int \frac{(\sin^n\theta - \sin\theta)^{\frac{1}{n}} \cos\theta}{\sin^{n+1}\theta} d\theta$

$= \int \frac{(t^n - t)^n dt}{t^{n+1}} \qquad (\text{Put } \sin\theta = t)$

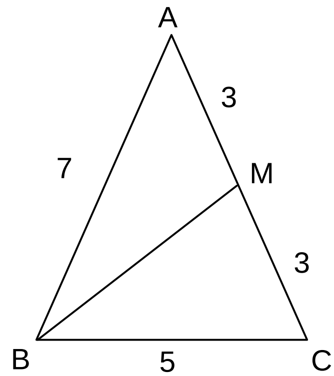
$= \int \frac{t\left(1 - \frac{1}{t^{n-1}}\right)^{\frac{1}{n}}}{t^{n+1}} dt = \int \frac{\left(1 - \frac{1}{t^{n-1}}\right)^{\frac{1}{n}}}{t^n} dt$

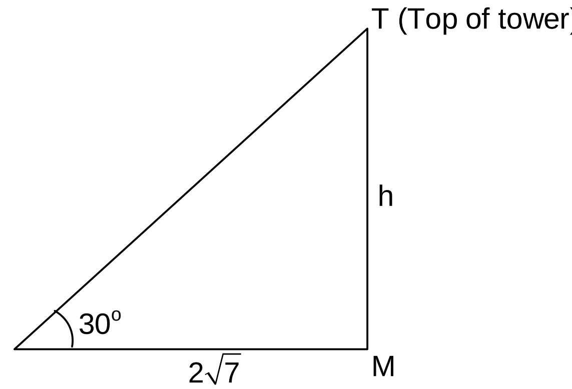
Put $1 - \frac{1}{t^{n-1}} = z \Rightarrow \frac{(n-1)}{t^n} dt = dz,$

$\Rightarrow I = \frac{1}{n-1} \int z^{\frac{1}{n}} dz = \frac{z^{\frac{1}{n}+1}}{\left(\frac{1}{n} + 1\right)(n-1)} + c = \frac{n\left(1 - t^{1-n}\right)^{n+1}}{n^2 - 1} + c$

$= \frac{n}{n^2 - 1}\left(1 - \frac{1}{\sin^{n-1}\theta}\right)^{\frac{h+1}{n}} + c$

85. Length of median $BM=\frac{1}{2}\sqrt{2\left(BC^2+BA^2\right)-\left(AC\right)^2}=\frac{1}{2}\sqrt{2\left(25+49\right)-36}$





$=\frac{1}{2}\sqrt{112}=\sqrt{\frac{112}{4}}=\sqrt{28}=2\sqrt{7}$

Let h be height of tower, given $\tan 30^o=\frac{h}{2\sqrt{h}}\Rightarrow h=2\sqrt{\frac{7}{3}}=\sqrt{\frac{28}{3}}$

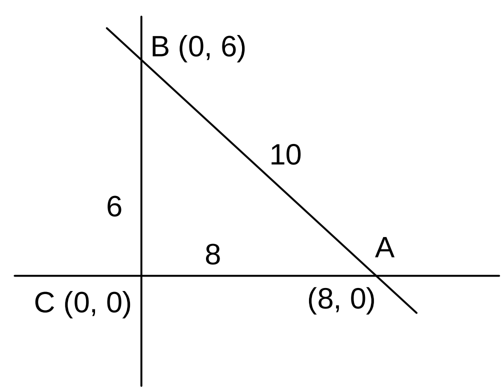
86. Hyperbola is $\frac{x^2}{5}-\frac{y^2}{4}=1$

Equation of its tangent in slope from is $y=mx\pm\sqrt{5m^2-4}$

Hence tangent with slope 1 is $y=x\pm 1$

87. $I=\left(\frac{ax_1+bx_2+cx_3}{a+b+c},\frac{ay_1+by_2+cy_2}{a+b+c}\right)$

$=\left(\frac{8\left(60\right)+6\left(0\right)+10\left(0\right)}{24},\frac{10\left(0\right)+6\left(8\right)+8\left(0\right)}{24}\right)=\left(2,\ 2\right)$



88. Two digit numbers of the form $7\lambda+2$ are 16, 23 ……………,93

Two digit numbers of the form $7\lambda+5$ are 12, 19,……………,96

Sum of all the above numbers equals to $\frac{12}{2}\left(16+93\right)+\frac{13}{2}\left(12+96\right)=654+702=1356$

89. $y = kx^2,\ x = ky^2$

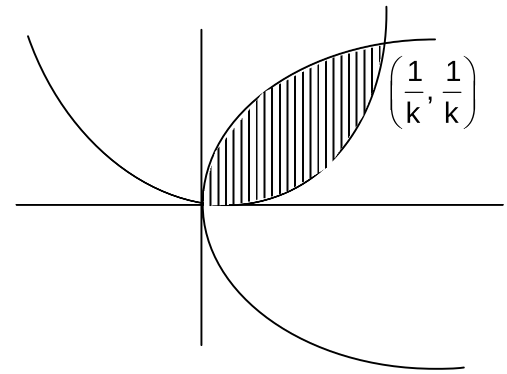
$\Rightarrow x = k\left(k^2x^4\right) \Rightarrow x = 0$ or $x^3 = \left(\frac{1}{k}\right)^3 \Rightarrow x = \frac{1}{k}, 0$

Point of intersection are $\left(\frac{1}{k}, \frac{1}{k}\right)$ and $(0,0)$

Area $= \int_0^{1/k}\left(\sqrt{\frac{x}{k}} - kx^1\right)dx = 1 \Rightarrow \left(\frac{1}{\sqrt{k}}\frac{x^{3/2}}{3/2} - \frac{kx^3}{3}\right)^{1/k}$

$= 1 \Rightarrow \frac{2}{3k^2} = 1 \Rightarrow k^2 = \frac{1}{3}$

$k = \frac{1}{\sqrt{3}}$



90. $P(n) = n^2 + 41$

$P(3) = 9 - 3 + 41 = 47$

$P(5) = 25 - 5 + 41 = 61$

Hence $P(3)$ and $P(5)$ are both prime