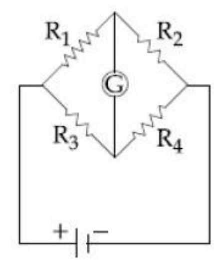
PART –A (PHYSICS)

1. The Wheatstone bridge shown in Fig. here, gets balanced when the carbon resistor used as R_1 has the colour code (Orange, Red, Brown). The resistors R_2 and R_4 are 80 Ω and 40 Ω, respectively. Assuming that the colour code for the carbon resistors gives their accurate values, the colour code for the carbon resistor, used as R_3 would be:



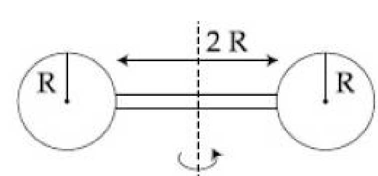
(A) Brown, Blue, Brown (B) Brown, Blue, Black
(C) Red, Green, Brown (D) Grey, Black, Brown

2. Consider the nuclear fission $Ne^{20} \rightarrow 2He^{4} + C^{12}$ .Given that the binding energy / nucleon of Ne^{20}, He^4 and C^{12} are, respectively, 8.03 MeV, 7.07 MeV, and 7.86 MeV, identify the correct statement:
(A) energy of 12.4 MeV will be supplied (B) 8.3 MeV energy will be released
(C) energy of 3.6 MeV will be released (D) energy of 11.9 MeV has to be supplied

3. A hoop and a solid cylinder of same mass and radius are made of a permanent magnetic material with their magnetic moment parallel to their respective axes. But the magnetic moment of hoop is twice of solid cylinder. They are placed in a uniform magnetic field in such a manner that their magnetic moments make a small angle with the field. If the oscillation periods of hoop and cylinder are T_h and T_C respectively, then :
(A) $T_h = T_C$ (B) $T_h = 2T_C$
(C) $T_h = 1.5T_C$ (D) $T_h = 0.5_C$

4. An unknown metal of mass 192 g heated to a temperature of 100°C was immersed into a brass calorimeter of mass 128 g containing 240 g of water at a temperature of 8.4°C. Calculate the specific heat of the unknown metal if water temperature stabilizes at 21.5°C. (Specific heat of brass is 394 J $kg^{-1} K^{-1}$)
(A) 458 J $kg^{-1} K^{-1}$ (B) 1232 J $kg^{-1} K^{-1}$
(C) 916 J $kg^{-1} K^{-1}$ (D) 654 J $kg^{-1} K^{-1}$

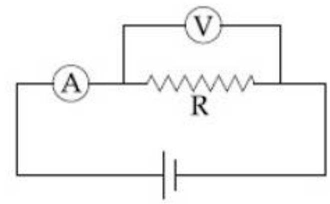
5. Two identical spherical balls of mass M and radius R each are stuck on two ends of a rod of length 2R and mass M (see figure). The moment of inertia of the system about the axis passing perpendicularly through the centre of the rod is :



(A) $\frac{137}{15}MR^2$ (B) $\frac{17}{15}MR^2$
(C) $\frac{209}{15}MR^2$ (D) $\frac{152}{15}MR^2$

6. The self produced emf of a coil is 25 volts. When the current in it is changed at uniform rate from 10 A to 25 A in 1 s, the change in the energy of the inductance is :
(A) 740 J (B) 437.5 J
(C) 540 J (D) 637.5 J

7. The actual value of resistance R, shown in the figure is 30 Ω. This is measured in an experiment as shown using the standard formula $R = \frac{V}{I}$, where V and I are the readings of the voltmeter and ammeter, respectively. If the measured value of R is 5% less, then the internal resistance of the voltmeter is :



(A) 600 Ω (B) 570 Ω
(C) 35 Ω (D) 350 Ω

8. At some location on earth the horizontal component of earth's magnetic field is 18×10^{-6} T. At this location, magnetic needle of length 0.12 m and pole strength 1.8 A m is suspended from its mid-point using a thread, it makes 45° angle with horizontal in equilibrium. To keep this needle horizontal, the vertical force that should be applied at one of its ends is :

(A) 3.6×10^{-5} N (B) 1.8×10^{-5} N
(C) 1.3×10^{-5} N (D) 6.5×10^{-5} N

9. Two vectors $\vec{A}$ and $\vec{B}$ have equal magnitudes. The magnitude of $(\vec{A} + \vec{B})$ is 'n' times the magnitude of $(\vec{A} - \vec{B})$. The angle between $\vec{A}$ and $\vec{B}$ is:

(A) $\cos^{-1}\left[\frac{n^2 - 1}{n^2 + 1}\right]$ (B) $\cos^{-1}\left[\frac{n-1}{n+1}\right]$
(C) $\sin^{-1}\left[\frac{n^2 - 1}{n^2 + 1}\right]$ (D) $\sin^{-1}\left[\frac{n-1}{n+1}\right]$

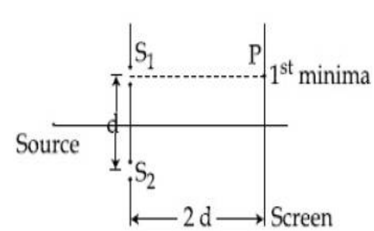
10. A metal plate of area 1×10^{-4} m^2 is illuminated by a radiation of intensity 16 mW/m^2. The work function of the metal is 5eV. The energy of the incident photons is 10 eV and only 10% of it produces photo electrons. The number of emitted photo electrons per second and their maximum energy, respectively, will be : [1 eV = 1.6×10^{-19} J]

(A) 10^{14} and 10 eV (B) 10^{12} and 5 eV
(C) 10^{11} and 5 eV (D) 10^{10} and 5 eV

11. A particle which is experiencing a force, given by $\vec{F} = 3\vec{i} - 12\vec{j}$, undergoes a displacement of $\vec{d} = 4\vec{i}$. If the particle had a kinetic energy of 3 J at the beginning of the displacement, what is its kinetic energy at the end of the displacement?

(A) 9 J (B) 12 K
(C) 10 J (D) 15 J

12. Consider a Young's double slit experiment as shown in figure. What should be the slit separation d in terms of wavelength λ such that the first minima occurs directly in front of the slit (S_1)?



(A) $\frac{\lambda}{2(\sqrt{5} - 2)}$ (B) $\frac{\lambda}{(\sqrt{5} - 2)}$
(C) $\frac{\lambda}{2(5 - \sqrt{2})}$ (D) $\frac{\lambda}{(5 - \sqrt{2})}$

13. The eye can be regarded as a single refracting surface. The radius of curvature of this surface is equal to that of cornea (7.8 mm). This surface separates two media of refractive indices 1 and 1.34. Calculate the distance from the refracting surface at which a parallel beam of light will come to focus.

(A) 1 cm
(B) 2 cm
(C) 4.0 cm
(D) 3.1 cm

14. A current of 2 mA was passed through an unknown resistor which dissipated a power of 4.4 W. Dissipated power when an ideal power supply of 11 V is connected across it is :

(A) 11×10^{-5} W
(B) 11×10^{-3} W
(C) 11×10^{-4} W
(D) 11×10^{5} W

15. The diameter and height of a cylinder are measured by a meter scale to be 12.6±0.1 cm and 34.2 ± 0.1 cm, respectively. What will be the value of its volume in appropriate significant figures?

(A) 4264±81 cm^3
(B) 4264±81.0 cm^3
(C) 4260±80 cm^3
(D) 4300±80 cm^3

16. For equal point charges Q each are placed in the xy plane at (0, 2), (4, 2), (4, –2) and (0, –2). The work required to put a fifth change Q at the origin of the coordinate system will be :

(A) $\frac{Q^2}{4\pi\epsilon_0}\left(1+\frac{1}{\sqrt{3}}\right)$
(B) $\frac{Q^2}{4\pi\epsilon_0}\left(1+\frac{1}{\sqrt{3}}\right)$
(C) $\frac{Q^2}{2\sqrt{2}\pi\epsilon_0}$
(D) $\frac{Q^2}{4\pi\epsilon_0}$

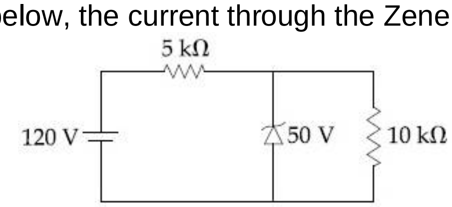
17. The modulation frequency of an AM radio station is 250 kHz, which is 10% of the carrier wave. If another AM station approaches you for license what broadcast frequency will you allot?

(A) 2750 kHz
(B) 2900 kHz
(C) 2250 kHz
(D) 2000 kHz

18. A closed organ pipe has a fundamental frequency of 1.5 kHz. The number of overtones that can be distinctly heard by a person with this organ pipe will be : (Assume that the highest frequency a person can hear is 20,000 Hz)

(A) 6
(B) 4
(C) 7
(D) 5

19. For the circuit shown below, the current through the Zener diode is :



(A) 9 mA
(B) 5 mA
(C) Zero
(D) 14 mA

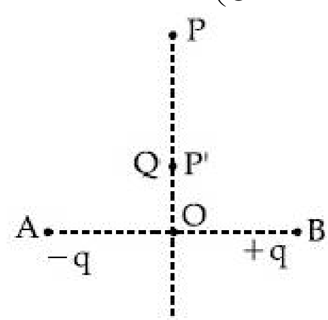
20. The electric field of a plane polarized electromagnetic wave in free space at time t = 0 is given by an expression

$\vec{E}(x, y) = 10\hat{j}\cos[(6x + 8z)]$

The magnetic field $\vec{B}$ (x, z, t) is given by : (c is the velocity of light)

(A) $\frac{1}{c}(6\hat{k} + 8\hat{i})\cos[(6x - 8z + 10ct)]$
(B) $\frac{1}{c}(6\hat{k} - 8\hat{i})\cos[(6x + 8z - 10ct)]$
(C) $\frac{1}{c}(6\hat{k} + 8\hat{i})\cos[(6x + 8z - 10ct)]$
(D) $\frac{1}{c}(6\hat{k} - 8\hat{i})\cos[(6x + 8z + 10ct)]$

21. Charges – q and + q located at A and B, respectively, constitute an electric dipole. Distance AB = 2a, O is the mid point of the dipole and OP is perpendicular to AB. A charge Q is placed at P where OP = y and y >> 2a. The charge Q experiences an electrostatic force F. If Q is now moved along the equatorial line to P′ such that $OP' = \left(\frac{y}{3}\right)$, the force on Q will be close to: $\left(\frac{y}{3} >> 2a\right)$



(A) 3F
(B) $\frac{F}{3}$
(C) 9F
(D) 27F

22. Two stars of masses 3×10^{31} kg each, and at distance 2×10^{11} m rotate in a plane about their common centre of mass O. A meteorite passes through O moving perpendicular to the star's rotation plane. In order to escape from the gravitational field of this double star, the minimum speed that meteorite should have at O is : (Take Gravitational constant G = 6.67×10^{-11} Nm^2 kg^{-2})

(A) 2.4×10^4 m/s
(B) 1.4×10^5 m/s
(C) 3.8×10^4 m/s
(D) 2.8×10^5 m/s

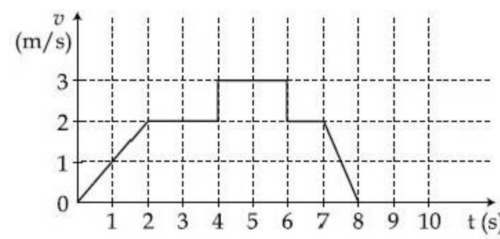
23. Half mole of an ideal monoatomic gas is heated at constant pressure of 1 atm from 20°C to 90°C. Work done by has is close to: (Gas constant R = 8.31 J/mol·K)

(A) 581 J
(B) 291 J
(C) 146 J
(D) 73 J

24. A parallel plate capacitor having capacitance 12 pF is charged by a battery to a potential difference of 10 V between its plates. The charging battery is now disconnected and a porcelain slab of dielectric constant 6.5 is slipped between the plates. The work done by the capacitor on the slab is :

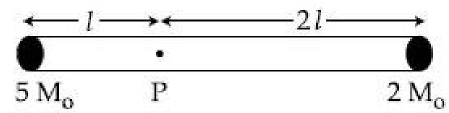
(A) 692 pJ
(B) 508 pJ
(C) 560 pJ
(D) 600 pJ

25. A particle starts from the origin at time t = 0 and moves along the positive x-axis. The graph of velocity with respect to time is shown in figure. What is the position of the particle at time t = 5s?



(A) 10 m (B) 6 m
(C) 3 m (D) 9 m

26. A rigid massless road of length 3l has tow masses attached at each end as shown in the figure. The rod is pivoted at point P on the horizontal axis (see figure). When released from initial horizontal position, its instantaneous angular acceleration will be :



(A) $\frac{g}{13l}$ (B) $\frac{g}{3l}$
(C) $\frac{g}{2l}$ (D) $\frac{7g}{3l}$

27. Two forces P and Q, of magnitude 2F and 3F, respectively, are at an angle θ with each other. If the force Q is doubled, then their resultant also gets doubled. Then, the angle θ is :
(A) 120° (B) 60°
(C) 90° (D) 30°

28. A cylindrical plastic bottle of negligible mass of filled with 310 ml of water and left floating in a pond with still water. If pressed downward slightly and released, it starts performing simple harmonic motion at angular frequency ω. If the radius of the bottle is 2.5 cm then ω is close to: (density of water = 10^3 kg/m^3)
(A) 3.75 rad s^{-1} (B) 1.25 rad s^{-1}
(C) 2.50 rad s^{-1} (D) 5.00 rad s^{-1}

29. A particle executes simple harmonic motion with an amplitude of 5 cm. When the particle is at 4 cm from the mean position, the magnitude of its velocity is SI units is equal to that of its acceleration. Then, its periodic time in seconds is :
(A) $\frac{4\pi}{3}$ (B) $\frac{3}{8}\pi$
(C) $\frac{8\pi}{3}$ (D) $\frac{7}{3}\pi$

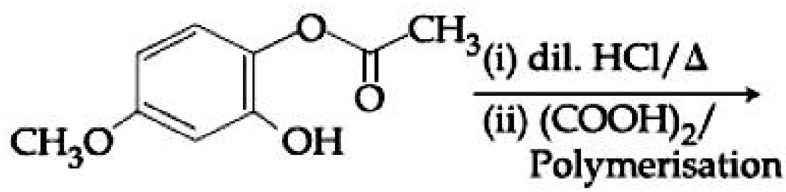
30. Two kg of a monoatomic gas is at a pressure of 4×10^4 N/m^2. The density of the gas is 8 kg/m^3. What is the order of energy of the gas due to its thermal motion?
(A) 10^3 J (B) 10^5 J
(C) 10^4 J (D) 10^6 J

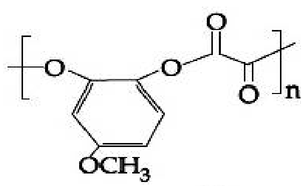
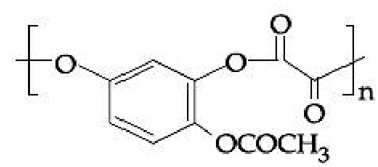
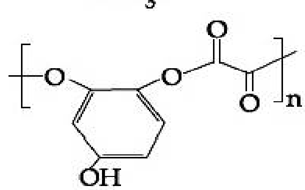
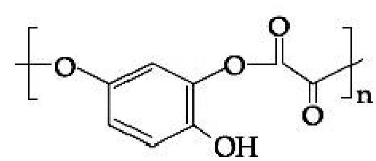
PART –B (CHEMISTRY)

31. The ground state energy of hydrogen atom is –13.6 eV. The energy of second excited state of He^+ ion in eV is:
(A) –54.4 (B) –3.4
(C) –6.04 (D) –27.2

32. Haemoglobin and gold sold are examples of:
(A) positively and negatively charged sols, respectively
(B) positively charged sols
(C) negatively charged sols
(D) negatively and positively charged sols, respectively

33. The major product of the following reaction is:



(A) 
(B) 
(C) 
(D) 

34. The amount of sugar ($C_{12}H_{22}O_{11}$) required to prepare 2 L of its 0.1 M aqueous solution is:
(A) 136.8 g (B) 17.1 g
(C) 68.4 g (D) 34.2 g

35. Among the following reactions of hydrogen with halogens, the one that requires a catalyst is:
(A) $H_2 + I_2 \rightarrow 2HI$ (B) $H_2 + Cl_2 \rightarrow 2HCl$
(C) $H_2 + Br_2 \rightarrow 2HBr$ (D) $H_2 + F_2 \rightarrow 2HF$

36. 5.1 g NH_4SH is introduced in 3.0 L evacuated flask at 327°C. 30% of the solid NH_4SH decomposed to NH_3 and H_2S as gases. The K_p of the reaction at 327°C is (R = 0.082 L atm mol^{-1} K^{-1}, Molar mass of S = 32 g mol^{-1}, molar mass of N = 14 g mol^{-1})
(A) 0.242×10^{-4} atm^2 (B) 1×10^{-4} atm^2
(C) 4.9×10^{-3} atm^2 (D) 0.242 atm^2

37. The reaction that is NOT involved in the ozone layer depletion mechanism in the stratosphere is :

(A) $CF_2Cl_2(g) \xrightarrow{uv} \dot{C}l(g) + \dot{C}F_2Cl(g)$

(B) $Cl\dot{O}(g) + O(g) \rightarrow \dot{C}l(g) + O_2(g)$

(C) $CH_4 + 2O_3 \rightarrow 3CH_2 = O + 3H_2O$

(D) $HOCl(g) \xrightarrow{hv} \dot{O}H(g) + \dot{C}l(g)$

38. In the cell $Pt(s)|H_2(g,1bar)|HCl(aq)|AgCl(s)|Ag(s)|Pt(s)$ the cell potential is 0.92 V when a 10^{-6} molal HCl solution is used. The standard electrode potential of $(AgCl/Ag,Cl^-)$ electrode is :

$\left\{\text{Given}, \frac{2.303RT}{F} = 0.06V \text{ at } 298K\right\}$

(A) 0.94 V
(B) 0.76 V
(C) 0.40 V
(D) 0.20 V

39. The 71st electron of an element X with an atomic number of 71 enters into the orbital:

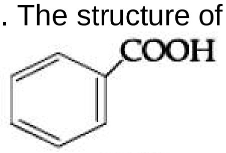
(A) 6p
(B) 4f
(C) 5d
(D) 6s

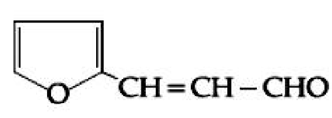
40. The correct match between item 'I' and item 'II' is:

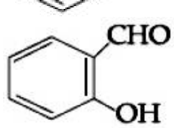
Item 'I' (compound)		**Item 'II' (reagent)**	
(a)	Lysine	(p)	1-naphtol
(b)	Furfural	(q)	Ninhydrin
(c)	Benzyl alcohol	(r)	$KMnO_4$
(d)	Styrene	(s)	Ceric ammonium nitrate

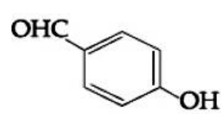
(A) (a)→(q); (b) →(p); (c) →(s); (d) →(r)
(B) (a)→(q); (b) →(p); (c) →(r); (d) →(s)
(C) (a)→(r); (b) →(p); (c) →(q); (d) →(s)
(D) (a)→(q); (b) →(r); (c) →(s); (d) →(p)

41. An aromatic compound 'A' having molecular formula $C_7H_6O_2$ on treating with aqueous ammonia and heating forms compound 'B'. The compound 'B' on reaction with molecular bromine and potassium hydroxide provides compound 'C' having molecular formula C_6H_7N. The structure of 'A' is :

(A) 

(B) 

(C) 

(D) 

42. The process with negative entropy change is:

(A) Dissociation of $CaSO_4(s)$ to $CaO(s)$ and $SO_3(g)$
(B) Sublimation of dry ice
(C) Dissolution of iodine in water
(D) Synthesis of ammonia from N_2 and H_2

43. An ideal gas undergoes isothermal compression from 5 m^3 to 1 m^3 against a constant external pressure of 4 NM^{-2}. Heat released in this process is used to increase the temperature of 1 mole of Al. If molar heat capacity of Al is 24 J $mol^{-1}K^{-1}$, the temperature of Al is increases by:

(A) $\frac{3}{2}$K (B) 2 K

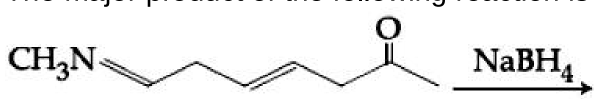
(C) $\frac{2}{3}$K (D) 1 K

44. Elevation in the boiling point for 1 molal solution of glucose is 2 K. The depression in the freezing point for 2 molal solution of glucose in the same solvent is 2 K. The relation between K_b and K_f is :

(A) $K_b = 1.5K_f$ (B) $K_b = K_f$

(C) $K_b = 0.5K_f$ (D) $K_b = 2K_f$

45. The major product of the following reaction is :



(A) 

(B) 

(C) 

(D) 

46. Sodium metal on dissolution in liquid ammonia gives a deep blue solution due to the formation of:

(A) sodium-ammonia complex (B) sodamide

(C) sodium ion-ammonia complex (D) ammoniated electrons

47. For an elementary chemical reaction, $A_2 \underset{k_{-1}}{\overset{k_1}{\rightleftharpoons}} 2A$, the expression for $\frac{d[A]}{dt}$ is

(A) $k_1[A_2] - k_{-1}[A]^2$ (B) $2k_1[A_2] - k_{-1}[A]^2$

(C) $k_1[A_2] + k_{-1}[A]^2$ (D) $2k_1[A_2] - 2k_{-1}[A]^2$

48. Which of the following tests cannot be used for identifying amino acids?

(A) Biuret test (B) Barfoed test

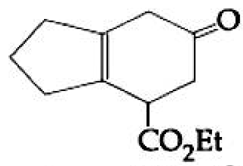
(C) Ninhydrin test (D) Xanthoproteic test

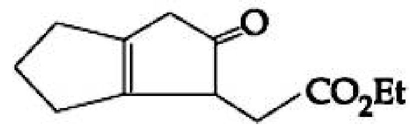
49. The difference in the number of unpaired electrons of a metal ion in its high-spin and low-spin octahedral complexes is two. The metal ion is :

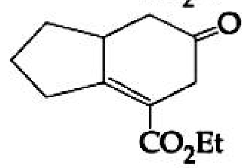
(A) Ni^{2+} (B) Fe^{2+}

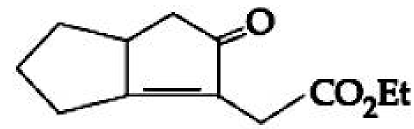
(C) Co^{2+} (D) Mn^{2+}

50. The major product obtained in the following reaction is:

(A) 

(B) 

(C) 

(D) 

51. The pair that contains two P – H bonds in each of the oxoacids is :

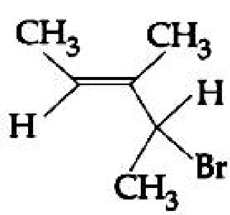
(A) $H_4P_2O_5$ and $H_4P_2O_6$
(B) H_3PO_2 and $H_4P_2O_5$
(C) H_3PO_3 and H_3PO_2
(D) $H_4P_2O_5$ and H_3PO_3

52. Which is the most suitable reagent for the following transformation?

$$CH_3-CH=CH-CH_2-\underset{}{\overset{OH}{\overset{|}{C}H}}-CH_3 \longrightarrow CH_3-CH=CH-CH_2CO_2H$$

(A) Tollen's reagent
(B) I_2 / NaOH
(C) CrO_2Cl_2 / Cs_2
(D) alkaline $KMnO_4$

53. What is the IUPAC name of the following compound?



(A) 3-Bromo-1, 2-dimethylbut-1-ene
(B) 3-Bromo-3-methyl-1, 2-dimethylprop-1-ene
(C) 2-Bromo-3-methylpent-3-ene
(D) 4-Bromo-3-methylpent-2-ene

54. The number of 2-centre-2-electron and 3-centre-2-electron bonds in B_2H_6, respectively, are :

(A) 2 and 1
(B) 4 and 2
(C) 2 and 2
(D) 2 and 4

55. In the reaction of oxalate with permanganate in acidic medium, the number of electrons involved in producing one molecule of CO_2 is :

(A) 1
(B) 10
(C) 2
(D) 5

56. A reaction of cobalt(III) chloride and ethylenediamine in a 1 : 2 mole ratio generates two isomeric products A (violet coloured) and B (green coloured). A can show optical activity, but, B is optically inactive. What type of isomers does A and B represent?

(A) Geometrical isomers
(B) Coordination isomers
(C) Linkage isomers
(D) Ionisation isomers

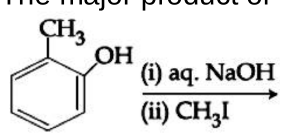
57. The electrolytes usually used in the electroplating of gold and silver, respectively, are :

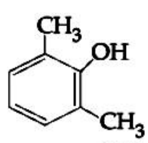
(A) $[Au(CN_2]^-$ and $[Ag(CN)_2]^-$
(B) $[Au(Cn)_2]^-$ and $[AgCl_2]^-$
(C) $[Au(OH)_4]^-$ and $[Ag(OH)_2]^-$
(D) $[Au(NH_3)_2]^+$ and $[Ag(CN)_2]^-$

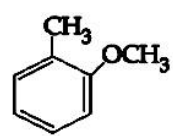
58. A compound of formula A_2B_3 has the hcp lattice. Which atom forms the hcp lattice and what fraction of tetrahedral voids is occupied by the other atoms :

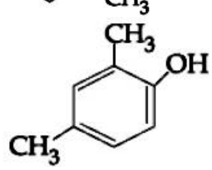
(A) hcp lattice - A, $\frac{2}{3}$ Tetrahedral voids - B
(B) hcp lattice - A, $\frac{1}{3}$ Tetrahedral voids - B
(C) hcp lattice - B, $\frac{2}{3}$ Tetrahedral voids - A
(D) hcp lattice - B, $\frac{1}{3}$ Tetrahedral voids – A

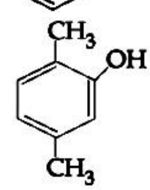
59. The major product of the following reaction is :



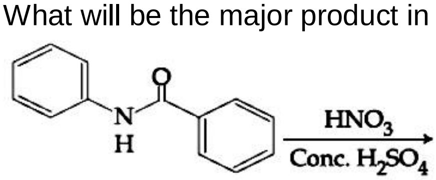
(A)


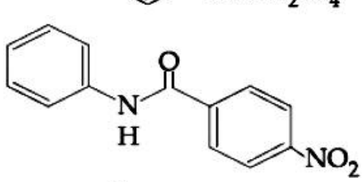
(B)


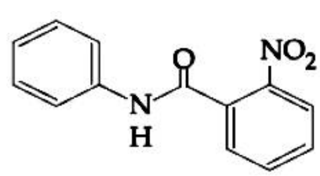
(C)


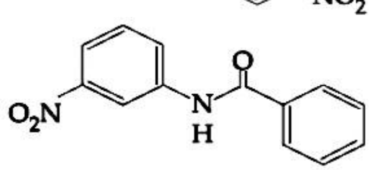
(D)


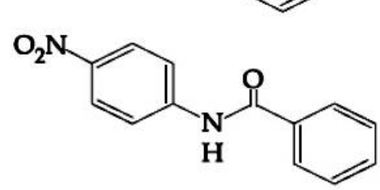
60. What will be the major product in the following mononitration reaction?



(A)


(B)


(C)


(D)


PART–C (MATHEMATICS)

61. The value of λ such that sum of the squares of the roots of quadratic equation, $x^2 + (3-\lambda)x + 2 = \lambda$ has the lest value is :

(A) $\frac{15}{8}$ (B) 1

(C) $\frac{4}{9}$ (D) 2

62. The value of $\cos\frac{\pi}{2^2}\cdot\cos\frac{\pi}{2^3}\cdot\ldots\cdot\cos\frac{\pi}{2^{10}}\cdot\sin\frac{\pi}{2^{10}}$ is :

(A) $\frac{1}{512}$ (B) $\frac{1}{1024}$

(C) $\frac{1}{256}$ (D) $\frac{1}{2}$

63. The curve amongst the family of curves represented by the differential equation, $(x^2 - y^2)dx + 2xy\ dy = 0$ which passes through (1, 1), is :

(A) a circle with centre on the x-axis
(B) an ellipse with major axis along the y-axis
(C) a circle with centre on the y-axis
(D) a hyperbola with transverse axis along the x-axis

64. Let f : (–1, 1)→R be a function defined by $f(x) = \max\left\{-|x|, -\sqrt{1-x^2}\right\}$. if K be the set of all points at which f is not differentiable, then K has exactly :

(A) five elements (B) one element

(C) three elements (D) two elements

65. The positive value of λ for which the co-efficient of x^2 in the expression $x^2\left(\sqrt{x}+\frac{\lambda}{x^2}\right)^{10}$ is 720, is :

(A) 4 (B) $2\sqrt{2}$

(C) $\sqrt{5}$ (D) 3

66. The tangent to the curve, $y = xe^{x^2}$ passing, through the point (1, e) also passes through the point :

(A) (2, 3e) (B) $\left(\frac{4}{3}, 2e\right)$

(C) $\left(\frac{5}{3}, 2e\right)$ (D) (3, 6e)

67. Let N be the set of natural numbers and two functions f and g be defined as f, g : N→N such that

$$f(n)=\begin{cases}\frac{n+1}{2} & \text{if } n \text{ is odd}\\ \frac{n}{2} & \text{if } n \text{ is even}\end{cases}$$

and $g(n)=n-(-1)^n$. Then fog is :

(A) onto but not one-one.
(B) one-one but not onto.
(C) both one-one and onto.
(D) neither one-one nor onto.

68. The number of values of $\theta\in(0,\pi)$ for which the system of linear equations

$x+3y+7z=0$

$-x+4y+7z=0$

$(\sin 3\theta)x+(\cos 2\theta)y+2z=0$ has a non-trivial solution, is :

(A) three
(B) two
(C) four
(D) one

69. Let $\vec{\alpha}=(\lambda-2)\,\vec{a}+\vec{b}$ and $\vec{\beta}=(4\lambda-2)\,\vec{a}+3\vec{b}$ be two given vectors where $\vec{a}$ and $\vec{b}$ are non collinear. The value of λ for which vectors $\vec{\alpha}$ and $\vec{\beta}$ are collinear, is :

(A) –4
(B) –3
(C) 4
(D) 3

70. Two sides of a parallelogram are along the lines, x + y = 3 and x – y + 3 = 0. If its diagonals intersect at (2, 4), then one of its vertex is :

(A) (3, 5)
(B) (2, 1)
(C) (2, 6)
(D) (3, 6)

71. If $\int_0^x f(t)\,dt = x^2+\int_x^1 t^2 f(t)\,dt$, then $f'\left(1/2\right)$ is :

(A) $\frac{24}{25}$
(B) $\frac{18}{25}$
(C) $\frac{4}{5}$
(D) $\frac{6}{25}$

72. Let $z=\left(\frac{\sqrt{3}}{2}+\frac{i}{2}\right)^5+\left(\frac{\sqrt{3}}{2}-\frac{i}{2}\right)^5$. If R(z) and I(z) respectively denote the real and imaginary parts of z, then :

(A) I(z) = 0
(B) R(z) > 0 and I(z) > 0
(C) R(z) < 0 an I(z) > 0
(D) R(z) = –3

73. If the probability of hitting a target by a shooter, in any shot, is $\frac{1}{3}$, then the minimum number of independent shots at the target required by him so that the probability of hitting the target at least once is greater than $\frac{5}{6}$, is :

(A) 3
(B) 6
(C) 5
(D) 4

74. If $\int x^5 e^{-4x^3}\, dx = \frac{1}{48} e^{-4x^3} f(x) + C$, where C is a constant of integration, then f(x) is equal to :

(A) $-2x^3 - 1$ (B) $-4x^3 - 1$

(C) $-2x^3 + 1$ (D) $4x^3 + 1$

75. If the area of an equilateral triangle inscribed in the circle, $x^2 + y^2 + 10x + 12y + c = 0$ is $27\sqrt{3}$ sq. units then c is equal to :

(A) 13 (B) 20

(C) –25 (D) 25

76. Consider the following three statements :

P : 5 is a prime number.

Q : 7 is a factor of 192.

R : L.C.M. of 5 and 7 is 35.

Then the truth value of which one of the following statements is true?

(A) $(\sim P) \vee (Q \wedge R)$ (B) $(P \wedge Q) \vee (\sim R)$

(C) $(\sim P) \wedge (\sim Q \wedge R)$ (D) $P \vee (\sim Q \wedge R)$

77. The length of the chord of the parabola $x^2 = 4y$ having equation $x - \sqrt{2}y + 4\sqrt{2} = 0$ is :

(A) $3\sqrt{2}$ (B) $2\sqrt{11}$

(C) $8\sqrt{2}$ (D) $6\sqrt{3}$

78. Let $A = \begin{bmatrix} 2 & b & 1 \\ b & b^2+1 & b \\ 1 & b & 2 \end{bmatrix}$ where b > 0. Then the minimum value of $\frac{\det(A)}{b}$ is :

(A) $2\sqrt{3}$ (B) $-2\sqrt{3}$

(C) $-\sqrt{3}$ (D) $\sqrt{3}$

79. Let $S = \left\{(x, y) \in R^2 : \frac{y^2}{1+r} - \frac{x^2}{1-r} = 1\right\}$, where $r \neq \pm 1$. Then S represents :

(A) a hyperbola whose eccentricity is $\frac{2}{\sqrt{1-r}}$, when $0 < r < 1$.

(B) an ellipse whose eccentricity is $\sqrt{\frac{2}{r+1}}$, when $r > 1$.

(C) a hyperbola whose eccentricity is $\frac{2}{\sqrt{r+1}}$, when $0 < r < 1$.

(D) an ellipse whose eccentricity is $\frac{1}{\sqrt{r+1}}$, when $r > 1$.

80. if $\sum_{r=0}^{25} \left\{ {}^{50}C_r \cdot {}^{50-r}C_{25-r} \right\} = K\left({}^{50}C_{25}\right)$, then K is equal to :

(A) $(25)^2$ (B) $2^{25} - 1$

(C) 2^{24} (D) 2^{25}

81. The plane which bisects the line segment joining the points (–3, –3, 4) and (3, 7, 6) at right angles, passes through which one of the following points?

(A) (–2, 3, 5) (B) (4, –1, 7)

(C) (2, 1, 3) (D) (4, 1, –2)

82. The value of $\cot\left(\sum_{n=1}^{19}\cot^{-1}\left(1+\sum_{p=1}^{n}2p\right)\right)$ is :

(A) $\frac{21}{19}$ (B) $\frac{19}{21}$

(C) $\frac{22}{23}$ (D) $\frac{23}{22}$

83. If mean and standard deviation of 5 observations x_1, x_2, x_3, x_4, x_5 are 10 and 3, respectively, then the variance of 6 observations $x_1, x_2, \ldots, x_3$ and –50 is equal to :

(A) 509.5 (B) 586.5

(C) 582.5 (D) 507.5

84. Let f be a differential function such that

$f'(x) = 7 - \frac{3}{4}\frac{f(x)}{x}, (x > 0)$ and $f(1) \neq 4$. Then $\lim_{x\to 0^+} x f\left(\frac{1}{x}\right)$:

(A) exists and equals $\frac{4}{7}$. (B) exists and equals 4.

(C) does not exist. (D) exists and equals 0.

85. Two vertices of a triangle are (0, 2) and (4, 3). If its orthocenter is at the origin, then its third vertex lies in which quadrant?

(A) third (B) second

(C) first (D) fourth

86. The value of $\int_{-\pi/2}^{\pi/2}\frac{dx}{[x]+[\sin x]+4}$, where [t] denotes the greatest integer less than or equal to t, is :

(A) $\frac{1}{12}(7\pi+5)$ (B) $\frac{1}{12}(7\pi-5)$

(C) $\frac{3}{20}(4\pi-3)$ (D) $\frac{3}{10}(4\pi-3)$

87. On which of the following lines lies the point of intersection of the line, $\frac{x-4}{2}=\frac{y-5}{2}=\frac{z-3}{1}$ and the plane, x + y + z = 2?

(A) $\frac{x+3}{3}=\frac{4-y}{3}=\frac{z+1}{-2}$ (B) $\frac{x-4}{1}=\frac{y-5}{1}=\frac{z-5}{-1}$

(C) $\frac{x-1}{1}=\frac{y-3}{2}=\frac{z+4}{-5}$ (D) $\frac{x-2}{2}=\frac{y-3}{2}=\frac{z+3}{3}$

88. Let $a_1, a_2, a_3, \ldots, a_{10}$ be in G.P. with $a_i > 0$ for $i = 1, 2, \ldots, 10$ and S be the set of pairs $(r, k), r, k \in N$ (the set of natural numbers) for which

$$\begin{vmatrix} \log_e a_1^r a_2^k & \log_e a_2^r a_3^k & \log_e a_3^r a_4^k \\ \log_e a_4^r a_5^k & \log_e a_5^r a_6^r & \log_e a_6^r a_7^k \\ \log_e a_7^r a_8^k & \log_e a_8^r a_9^k & \log_e a_9^r a_{10}^k \end{vmatrix} = 0$$

Then the number of elements in S, is :

(A) 4
(B) infinitely many
(C) 2
(D) 10

89. With the usual notation, in ΔABC, if ∠A + ∠B = 120°, $a = \sqrt{3} - 1$, then the ratio ∠A : ∠B, is :

(A) 7 : 1
(B) 5 : 3
(C) 9 : 7
(D) 3 : 1

90. A helicopter is flying along the curve given by $y - x^{3/2} = 7, (x \geq 0)$. A solider positioned at the point $\left(\frac{1}{2}, 7\right)$ wants to shoot down the helicopter when it is nearest to him. Then this nearest distance is :

(A) $\frac{\sqrt{5}}{6}$
(B) $\frac{1}{3}\sqrt{\frac{7}{3}}$
(C) $\frac{1}{6}\sqrt{\frac{7}{3}}$
(D) $\frac{1}{2}$

HINTS AND SOLUTIONS
PART A – PHYSICS

1. $R_1 = 32 \times 10 = 320\ \Omega$

$R_3 = \frac{R_y}{R_2} \times R_1 = \frac{40 \times 320}{80} = 160\ \Omega$

$\therefore$ Colour code of R_3 be Brown, Blue, Brown.

2. $Q = (B.E.)_R - (B.E.)_P$

$= 20 \times 8.03 - (8 \times 7.07 + 12 \times 7.86)$

$= 160.6 - (56.56 + 94.32)$

$\therefore$ Q = +9.72 meV

9.72 MeV released.

3. $\because \quad T = 2\pi\sqrt{\frac{I}{\mu B}}$

$\frac{T_h}{T_c} = \sqrt{\frac{I_R}{I_c} \times \frac{\mu_c}{\mu_h}}$

$= \sqrt{2 \times \frac{1}{2}} = 1$

$T_h = T_c$

4. Heat loss = Heat gain

$192 \times S(100 - 21.5) = (128 \times 0.394 + 240 \times 4.2)(21.5 - 8.4)$

$192 \times 78.5 \times S = 1058.432 \times 13.1$

$S = 0.91995$ J/g K^{-1}

$S = 919.95$ J/kg K^{-1}

5. $I = 2\left[\frac{2}{5}MR^2 + M4R^2\right] + M\frac{4R^2}{12}$

$= MR^2\left[\frac{1}{3} + \frac{4}{5} + 8\right] = \frac{137}{15}MR^2$

6. $\frac{L\Delta T}{\Delta t} = 25$

$\Rightarrow \quad L = \frac{25 \times 1}{15} = \frac{5}{3}$

$\Delta U = \frac{1}{2}L\left(I_f^2 - I_1^2\right) = \frac{1}{2} \times \frac{5}{3} \times (25^2 - 10^2)$

$= \frac{5}{6} \times 525 = 4.7.5$ J

7. $\frac{30\,R}{R+30} = 30 \times 00.95$

$\Rightarrow$ R = 570 Ω

8. τ = F × 0.06 = 1.8 × 0.012 × 18 × 10^{-6}

F = 6.48 × 10^{-5}

9. $\left|\vec{A}+\vec{B}\right| = n\left|\vec{A}-\vec{B}\right|$

$\Rightarrow$ $A^2 + B^2 + 2AB\cos\theta$

$= n^2(A^2 + B^2 - 2AB\cos\theta)$

$\Rightarrow$ $\cos\theta(1+n^2) = \frac{2a^2(n^2-1)}{2a^2}$ $\quad [A = B = a$

$\cos\theta = \frac{n^2-1}{n^2+1}$

10. $n = \frac{16\times10^{-3}\times10^{-4}}{10\times10\times10^{-19}} \simeq 1.6\times10^{11}$

$(K.E.)_{max}$ = (10 – 5) eV = 5eV

11. K.E. – 3 = $\vec{F}\cdot\vec{d}$

K.E. = 3 + $(3\hat{i}-12\hat{j})\times(4\hat{i})$

K.E. = 3 + 12 = 15 J

12. $\sqrt{(2d)^2+(d)^2} - 2d = \frac{\lambda}{2}$

$\Rightarrow$ $(\sqrt{5}-2)d = \frac{\lambda}{2}$

$\Rightarrow$ $d = \frac{\lambda}{2(\sqrt{5}-2)}$

13. $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2-\mu_1}{R}$

$\Rightarrow$ $\frac{1.34}{v} - \frac{1}{-\infty} = \frac{0.34}{7.8}$

$\Rightarrow$ $v = \frac{1.34\times7.8}{0.34}$nm = 3.074 cm

14. $R = \frac{P}{I^2} = \frac{4.4}{4\times10^{-6}}\,\Omega$

$P' = \frac{V^2}{R} = \frac{11\times11\times4\times10^{-6}}{4.4}$ W

= 11 × 10^{-5} W

15. $v = \pi R^2 h = \frac{\pi}{4} D^2 h$

$= 4260\ cm^2$

$\because \quad \frac{\Delta v}{v} = 2\frac{\Delta D}{D} + \frac{\Delta h}{h}$

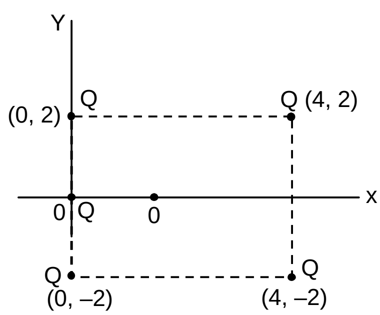
$= \left(2 \times \frac{0.1}{12.6} + \frac{0.1}{34.2}\right) v$

$= \frac{2x426}{12.6} + \frac{426}{34.2}$

$= 67.61 + 12.459 = 80.075$

$\therefore \quad v = 4260 \pm 80\ cm^3$

16.



$W = VQ = \frac{1}{4\pi\varepsilon_0} Q^2 \left[\frac{1}{2} + \frac{1}{2} + \frac{2}{2\sqrt{5}}\right]$

$\therefore \quad \frac{Q^2}{4\pi\varepsilon_0}\left[1 + \frac{1}{\sqrt{5}}\right]$

17. The interval between two carrier frequencies should be at least two times of AM frequency.

18. $\frac{250\ kHz}{1.5\ kHz} = 13.33$

$\therefore$ Possible hormones

1, 3, 5, 7, 9, 11, 13

i.e 6.

19. $i_{10\,k} = \frac{50}{10k} = 5\ mA$

$i_{5\,k} = \frac{120 - 50}{5k}$ = 14 mA

$i_2 = (14 - 5)\ mA$ = 9 mA

20. $\vec{E} = 10\hat{j} \cos(6x + 8z - 10ct)$

$B_o = \frac{E_o}{C} = \frac{10}{C}$

W = 10 C

$\because \quad \hat{E} \times \hat{B} = \hat{C}$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 1 & 0 \\ B_x & B_y & B_z \end{vmatrix} = \frac{6\hat{i}+8\hat{j}}{10}$$

$$\Rightarrow \quad B_z\hat{i} - 0\hat{j} - B_x\hat{k} = \frac{3}{5}\hat{i} + \frac{4}{5}\hat{j}$$

$$B_z = \frac{3}{5},\ B_y = 0,\ B_z = \frac{4}{5}$$

$$\therefore \quad \vec{B} = \frac{1}{C}(-8\hat{i} + 6\hat{k}) \cos(6x + 8z + 10\,ct)$$

21. $\because \quad F \propto \frac{1}{r^3}$

Required force = 27 F.

22. $\frac{1}{2}mv^2 + \frac{2(-GMm)}{r} = 0$

$V^2 = \frac{4GM}{r} = \frac{4 \times 6.67 \times 10^{-11} \times 3 \times 10^{31}}{2 \times 10^{11}}$

$V = 20\sqrt{2} \times 10^4$ m/s

= 2.828×10^5 m/s

23. W = $nR\Delta T$

= $\frac{1}{2} \times 8.31 \times 70$

= 290.85 J

24. $W = \frac{Q^2}{2c} - \frac{Q^2}{2ck}$

= $\frac{Q^2}{2c}\left[1 - \frac{1}{k}\right]$

= $\frac{1}{2} \times 12 \times 100\ \text{pJ}\left(1 - \frac{1}{6.5}\right)$

= $\frac{12 \times 100 \times 11}{2 \times 13}$pJ = 507.69 pJ

25. $r_{t=5}$ = area

= $\left(\frac{1}{2} \times 2 \times 2 + 2 \times 2 + 3 \times 1\right)$m

= (2 + 4 + 3) m

= 9 m.

26. $\alpha = \frac{\tau}{I}$

$= \frac{5M_o g\ell - 4M_o g\ell}{5M_o\ell^2 + 2M_o 4\ell^2}$

$= \frac{M_o g\ell}{13 M_o \ell^2}$

$= \frac{g}{13\,\ell}$

27. $2\left|\vec{P} + \vec{Q}\right| = \left|\vec{P} + 2\vec{Q}\right|$

$\Rightarrow$ 13 + 12 cos θ = 10 + 6 cos θ

$\cos = -\frac{1}{2}$

θ = 120°.

28. $A\rho g x = F_{restoring}$

$\pi r^2 \rho g x = n\omega^2 x$

$\therefore \quad \omega = \sqrt{\frac{\pi r^2 \rho g}{\rho v}}$

$= r\sqrt{\frac{\pi g}{v}} = 2.5 \times 10^{-2}\sqrt{\frac{3.14 \times 10}{310 \times 10^{-6}}}$

$= 2.5 \times 10^{-2} \times 10^2\sqrt{10}$

$\omega = 2.5 \times \sqrt{10}$

$\therefore \quad f = \frac{2.5 \times \sqrt{10}}{2\pi} = 1.25$

29. $|v_4| = |a_4|$

$\Rightarrow \quad \left(w\sqrt{A^2 - x^2}\right)_4 = (w^2 x)_4$

$\Rightarrow \quad w\sqrt{25 - 16} = w^2 \times 4$

$\Rightarrow \quad w = \frac{3}{4}$

$T = \frac{2\pi}{w} = 2\pi\frac{4}{3} = \frac{8\pi}{3}$

30. $E = \frac{1}{2} M V_m^2$

$= \frac{1}{2} \times 2 \times \left(\frac{3P}{\rho}\right)$

$= \frac{3 \times 4 \times 10^4}{8} = 1.5 \times 10^4$ J

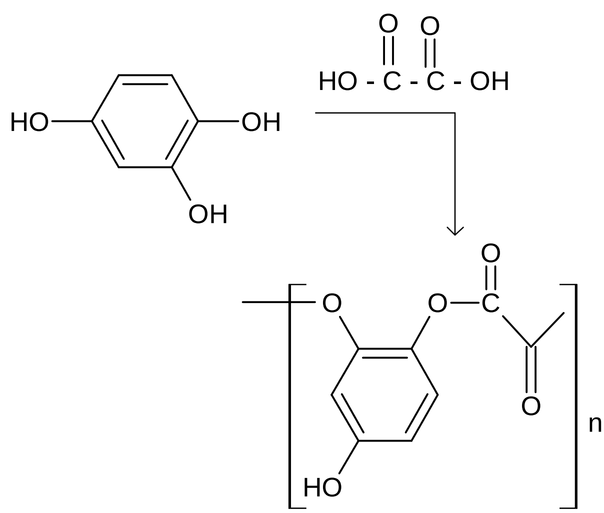
PART B – CHEMISTRY

31. $E = -13.6\frac{n^2}{z^2}eV$

$E_{He}^{+} = -13.6\frac{4}{9}$

32. Fact based

33.



34. $0.1 = \frac{n_{C_{12}H_{22}O_{11}}}{2}$

$n_{C_{12}H_{22}O_{11}} = 0.2$

$Wt_{C_{12}H_{22}O_{11}} = 0.2 \times 342 = 68.4$

35. First reaction will be requiring a catalyst among halogens oxidizing power decrease down the group.

36. $NH_4HS(s) \rightleftharpoons NH_3(g) + H_2S(g)$

5.1 g

0.1 g mol – 0.03 0.03 mol 0.03 mol

V = 3L, T = 327ºC $\frac{0.98}{2}$ $\frac{0.98}{2}$

$K_P = P_{NH_3}P_{H_2S}$ PV = nRT

$K_P = \frac{0.98}{2} \times \frac{0.98}{2}$ P × 3 = 0.06 × 0.0821 × 600

$P = \frac{0.06 \times 0.0821 \times 200}{3}$

K_P = 0.243 P = 0.98

37. CH_4 is not present in stratosphere.

38. $E = E^o - 0.06 \log \frac{[H^+][Cl^-]}{[H_2]^{1/2}}$

$0.92 = E^o - 0.06 \log \frac{10^{-6}.10^{-6}}{1^{1/2}}$

$0.92 = E^o - 0.06 \log 10^{-12}$

$E^o + 0.06 \times 12$

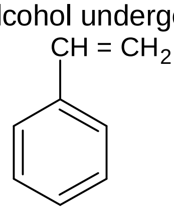
$E^o = 0.92 - 0.06 \times 12$

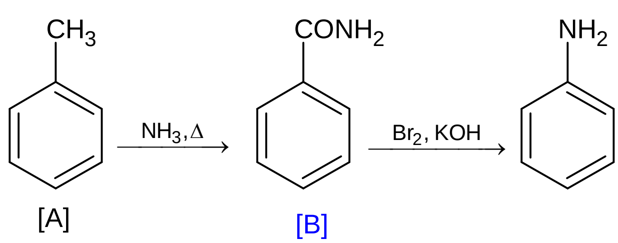
$E^o = 0.92 - 0.72$

$E^o_{Ag} / AgCl = 0.20$

39. The electron configuration is $[Xe]4f^{14}5d^16s^2$

40. Lysine is an amino acid nindydrin test is used for amino acids.
Furfural reacts with 1-napthol to give violet colouration.
Benzyl alcohol undergoes reaction with ceric ammonium nitrate to give red colouration.

Styrene , discharges color of $KMnO_4$

41. 

42. $CaSO_4(s) \longrightarrow CaO(s) + SO_3(g)$

$CO_2(s) \longrightarrow CO_2(g)$

$I_2 \longrightarrow I_2(aq)$

$N_2(g) + 3H_2(g) \longrightarrow 2NH_3(g)$

$N_2(g) + 3H_2 \rightleftharpoons 2NH_3(g)$

$\Delta S = 2\Delta S_{NH_3} - [\Delta S_{N_2} + 3\Delta S_{H_2}]$

There is decrease in number of moles of NH_3 entropy is decreasing.

43. $q = P\Delta V$

$q = 16$

$C_P = 24$

$C_P = \frac{qP}{\Delta T}$

$\Delta T = \frac{16}{24} K = \frac{2}{3} K$

44. $\Delta T_b = K_b m$

$\Delta T_b = K_b \times 1 \Rightarrow 2 = K_b$

$\Delta T_f = K_f m$

$2 = 2K_f \ = K_f$

$\frac{K_f}{K_b} = \frac{1}{2}$

45. $NaBH_4$ reduces both carbonyl group and imine.

46. $Na(s) + (x+y)NH_3 \longrightarrow Na^+(NH_3)_x + \underset{\text{Blue colour ammoniated electrons}}{e^-(NH_3)_y}$

47. $\frac{1}{2}\frac{d[A]}{dt} = -K_1[A]_2 + K_{-1}[A]^2$

$\frac{d[A]}{dt} = 2K_1[A_2] - 2K_{-1}[A]^2$

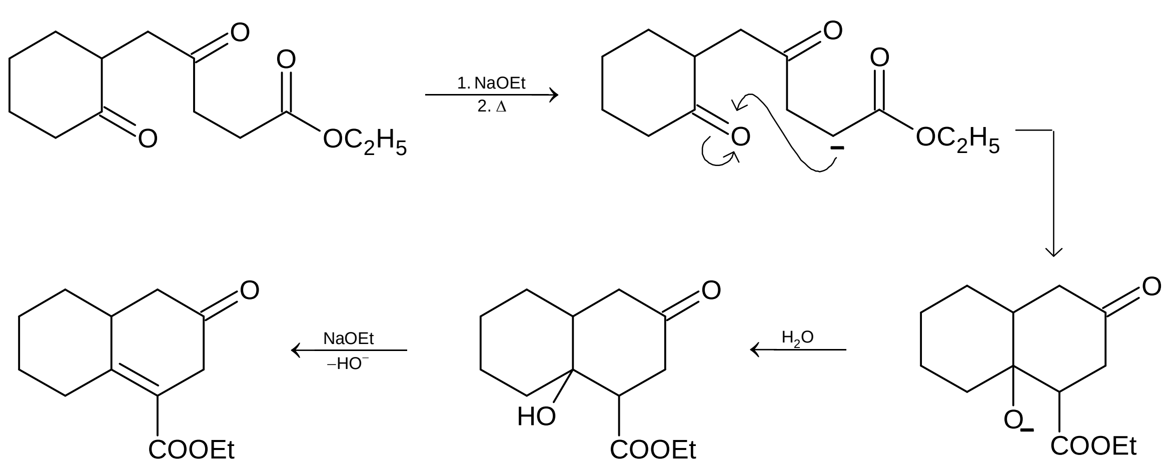
48. Barfoed test is used to detect monosaccirides.

49. Co^{2+} high spin $t_{2g}^5 e_g^2$ '3' unpaired electrons

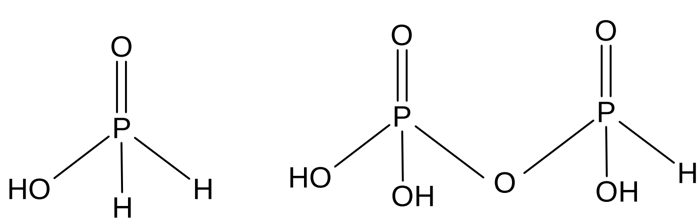
Co^{2+} low spin $t_{2g}^6 e_g^1$ '1' unpaired electron

Difference is 3 – 1 = 2

50.



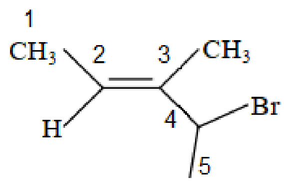
51.



H_3PO_2 $H_4P_2O_5$

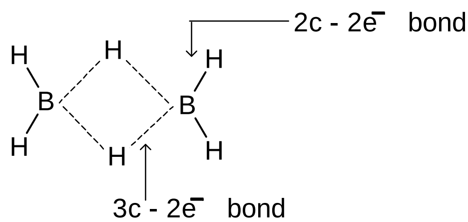
52. Iodoform reaction can be used for this transformation.

53.



4-Bromo-3-methyl pentane

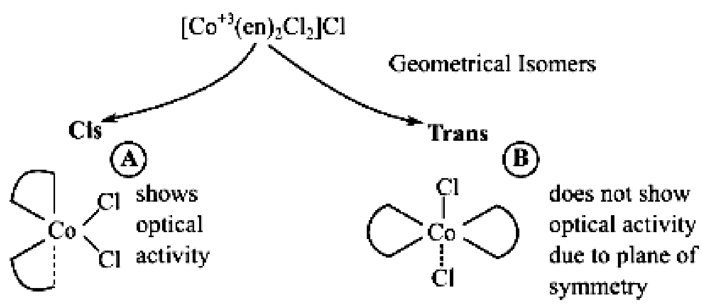
54.



55. $5C_2O_4^{2-} + 2KMnO_4 + H^+ \longrightarrow 10CO_2 + 2Mn^{2+} + H_2O$

After balancing 10 gain or loss of electron.

56. Cobalt(III) chloride on reaction with ethylenediemine in ratio 1 : 2 2 isomeric products complexes A and B



57. $[Au(CN_2]^-$ and $[Ag(CN)_2]^-$ both are soluble complexes.

58. Formula of the compounds

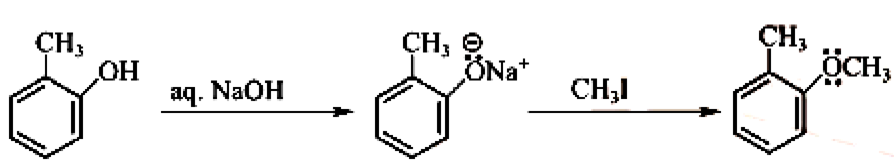
No. of octahedral voids are equal to the number of atoms forming lattice

A occupy octahedral void i.e. 2/3 of them

B forms crystal lattice

$A_{2/3}B \Rightarrow A_2B_3$

59.



60. 'EAS' first ring is activated and second is deactivated NO_2^+ attack at para position of activated ring.

PART C – MATHEMATICS

61. $\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta$

$=(3-\lambda)^2-2(2-\lambda)$

$=\lambda^2+9-6\lambda-4+2\lambda$

$=\lambda^2-4\lambda+5$

$\therefore$ For least value $\lambda=2$

62. Using formula $\frac{\sin 2^n A}{2^n \sin A}=\cos A\cos 2A\cos 2^2 A..........\cos 2^{n-1}A$

63. $x^2dx+2xydy-y^2dx=0$

$x^2dx=y^2dx-2xy\,dy$

$\left(x^2-y^2\right)dx+2xy\,dy=0$

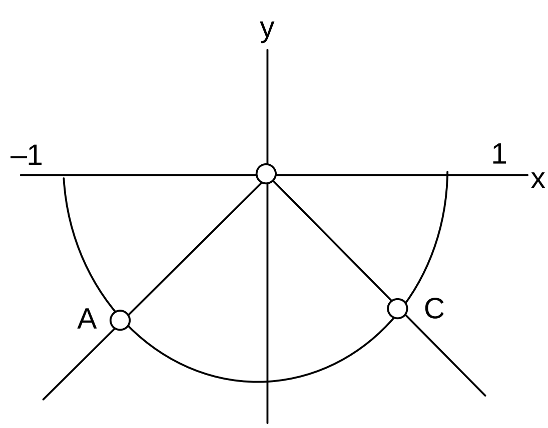
$dx=-\left(\frac{x.2y\,dy-y^2dx}{x^2}\right)$

Integrals

$x=-\frac{y^2}{x}+c$

$x^2+y^2=cx$

64. A, B, C are sharp edges



65. $x^2\left(\sqrt{x}+\frac{\lambda}{x^2}\right)^{10}$

Consider constant term

${}^{10}C_r\left(\sqrt{x}\right)^{10-r}\left(\frac{\lambda}{x^2}\right)^r$

$\frac{10-r}{2}-2r=0$

$10-5r=0$

$r=2$

$\Rightarrow {}^{10}C_2\times\lambda^2=720\Rightarrow\lambda=4$

66. $y = xe^{x^2}$

(1, e) lies on this

Now $\frac{dy}{dx} = xe^{x^2}.2x + e^{x^2}.1$

Put x = 1

$m = 2e + e = 3e$

Equation of tangent at (1, e)

$y - e = 3e(x - 1)$

$y - e = 3ex - 3e$

$y = 3ex - 2e$

$\left(\frac{4}{3}, 2e\right)$ satisfies it

$\therefore$ Answer is B

67. $\left.\begin{matrix} f(g(1)) = 1 \\ f(g(2)) = 1 \end{matrix}\right\}$ Many one

$f(g(2k)) = k$

$f(g(2k+1)) = k + 1$

$\therefore$ Onto

68. $\begin{vmatrix} \sin 3\theta & -1 & 1 \\ \cos 2\theta & 4 & 3 \\ 2 & 7 & 7 \end{vmatrix} = 0$

$7\sin 3\theta + 14\cos 2\theta - 14 = 0$

$\sin 3\theta + 2\cos 2\theta - 2 = 0, \sin\theta = \frac{1}{2}$

69. $\overline{\alpha} = (\lambda - 2)\vec{a} + \vec{b}$

$\vec{\beta} = (4\lambda - 2)\vec{a} + 3\vec{b}$

$\vec{\alpha}$ and $\vec{\beta}$ are collinear

$\begin{vmatrix} \lambda - 2 & 1 \\ 4\lambda - 2 & 3 \end{vmatrix} = 0$

$3\lambda - 6 - 4\lambda + 2 = 0$

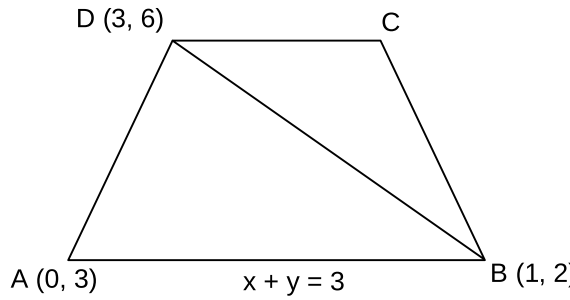
$-\lambda - 4 = 0$

$\lambda = -4$

70. Intersection point is A (0, 3)

$M = (4, 6)$

$B \Rightarrow (1, 2), D \rightarrow (3, 6)$



71. Differentiability we get $f(x) = 2x - x^2 f(x)$

$$f(x) = \frac{2x}{1+x^2} \Rightarrow f''(x) = 2\frac{(1-x^2)}{(1+x^2)^2}$$

$$f'\left(\frac{1}{2}\right) = \frac{24}{25}$$

72. $$\left(\frac{\sqrt{3}}{2} + \frac{i}{2}\right)^5 = \left(e^{i\frac{\pi}{b}}\right)^5 = e^{i5\pi/6}$$

$$\therefore \quad z = 2\cos\frac{5\pi}{6} = 2\left(-\frac{\sqrt{3}}{2}\right) = -\sqrt{3}$$

73. $$1 - \left(\frac{2}{3}\right)^n > \frac{5}{6}$$

$$\left(\frac{2}{3}\right)^n < \frac{1}{6}$$

$$\Rightarrow n = 5$$

74. $$\int x^5 . e^{-4x^3} dx$$

$$= \int x^2 . x^3 e^{-4x^3}\, dx$$

$$-4x^3 = t$$

$$-12x^2 dx = dt$$

$$= \frac{-1}{12}\int -\frac{t}{4} e^t\, dt$$

$$= \frac{1}{48}\int t\, e^t\, dt$$

$$= \frac{1}{48} te^t - 1.e^t + c$$

$$= \frac{1}{48} e^{-4x^3} . (-4x^3) - e^{-4x^3} + c$$

75. $$r = \sqrt{25 + 36 - c} = \sqrt{36}$$

$$c = 25$$

76. P is True

Q is False

R is True

$\therefore \sim Q$ is True

$\sim Q \wedge R$ is True

$\therefore P \vee (\sim Q \wedge R)$ is True.

77. $x = \sqrt{2}y - 4\sqrt{2}$

$x^2 = 4y$

Solving we get point of intersection

$A\left(-2\sqrt{2}, 2\right), B\left(4\sqrt{2}, 8\right)$

$\therefore AB = \sqrt{\left(6\sqrt{2}\right)^2 + 6^2} = 6\sqrt{3}$

78. $\text{Det } A = b^2 + 3$

$\frac{\det A}{b} = b + \frac{3}{b}$

$\therefore$ Least value $= 2\sqrt{3}$

79. $\frac{y^2}{1+r} - \frac{x^2}{1-r} = 1$

$r > 1 \quad \Rightarrow$ ellipse

$e = \sqrt{1 - \left(\frac{r-1}{r+1}\right)} = \sqrt{\frac{2}{r+1}}$

80. $\sum_{r=1}^{25} \frac{\lfloor 50}{\lfloor r \lfloor 50-r} \times \frac{\lfloor 50-r}{\lfloor 25-r \lfloor 25}$

$= \sum_{r=1}^{25} \frac{\lfloor 50}{\lfloor r \lfloor 25-r \lfloor 25}$

$= \frac{\lfloor 50}{\lfloor 25} \sum_{r=1}^{25} \frac{1}{\lfloor r \lfloor 25-r}$

$= \frac{\lfloor 50}{\lfloor 25 \lfloor 25} \sum_{r=1}^{25} {}^{25}C_r = {}^{50}C_{25}\left(2^{25} - 1\right)$

81. A (–3, 3, 4), B (3, 7, 6)

Mid point $\Rightarrow (0, 2, 5)$

$\vec{n} = \overrightarrow{AB} = 6\hat{i} + 10\hat{j} + 2\hat{k}$

Equation of plane $\vec{r}.\vec{n} = \vec{a}.\vec{n}$

$\vec{r}.\left(6\hat{i} + 10\hat{j} + 2\hat{k}\right) = \left(0\hat{i} + 2\hat{j} + 5\hat{k}\right).\left(6\hat{i} + 10\hat{j} + 2\hat{k}\right)$

$3x + 5y + z = 15$

(4, 1, –2) satisfies it

$\therefore$ Answer is D

82. $\cot\left[\sum_{n=1}^{19}\cot^{-1}\left(1+\sum_{p=1}^{n}2p\right)\right]$

$=\cot\left[\sum_{n=1}^{19}\cot^{-1}\left(1+n^2+n\right)\right]$

$=\cot\left[\sum_{n=1}^{19}\tan^{-1}\left(\frac{1}{1+n^2+n}\right)\right]$

$=\cot\left[\sum_{n=1}^{19}\tan^{-1}(n+1)-\tan^{-1}1\right]$

$=\cot\left[\tan^{-1}20-\tan^{-1}1\right]$

$=\cot\left(\tan^{-1}\frac{19}{21}\right)$

$\Rightarrow\frac{21}{19}$

83. $\sum x=50$

$(3)^2=\frac{1}{5}\left(ex^2-\frac{(ex)^2}{5}\right)$

$9=\frac{1}{5}\left(\sum x^2-\frac{2500}{5}\right)$

$\therefore\sum x^2=545$

New variable $=\frac{1}{6}\left(3045-\frac{0}{6}\right)=507.5$

84. $f'(x)=7-\frac{3}{4}.\frac{f(x)}{x},\ x>0$

$\therefore f'(x)+\frac{3}{4x}f(x)=7$ (Linear)

$f(x).e^{\int\frac{3}{4x}dx}=\int 7.e^{\int\frac{3}{4x}dx}+c$

$f(x).x^{3/4}=\int 7.x^{3/4}+c$

$=7\frac{x^{7/4}}{\frac{7}{4}}+c$

$\therefore\ f(x)=4x+cx^{-3/4}$

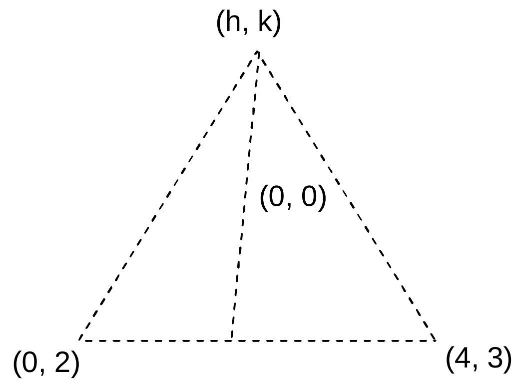
$\therefore f\left(\frac{1}{x}\right)=\frac{4}{x}+cx^{3/4}$

$\therefore\ \underset{x\to0^+}{\text{Lt}}\ xf\left(\frac{1}{x}\right)=\underset{x\to0^+}{\text{Lt}}\ 4+c\,x^{7/4}=4$

85. $\frac{k-3}{h-4}=0 \qquad k=3$

$\frac{k}{h}=-\frac{4-0}{3-2} \qquad -4h=k$

$h=\frac{-3}{4}$



86. $\int_{-\pi/2}^{\pi/2}\frac{dx}{[x]+[\sin x]+4}$

$=\int_{-\pi/2}^{0}\frac{dx}{[x]+-1+4}+\int_{0}^{\frac{\pi}{2}}\frac{dx}{[x]+4}$

$=\int_{-\pi/2}^{-1}\frac{dx}{-2-1+4}+\int_{-1}^{0}\frac{dx}{-1-1+4}+\int_{0}^{1}\frac{dx}{4}+\int_{1}^{\pi/2}\frac{dx}{1+4}$

$=-1+\frac{\pi}{2}+2+\frac{1}{4}+\frac{1}{5}\left(\frac{\pi}{2}-1\right)$

$=3\frac{\pi}{5}-\frac{9}{20}$

87. Put $(2\lambda+4,\ 2\lambda+5,\ \lambda+3)$ in $x+y+z=2$

$2\lambda+4+2\lambda+5+\lambda+3=2$

$5\lambda=-10 \qquad \lambda=-2$

$P(0,1,1)$

Now put in options

Answer is C

88. For any value of r determinant is zero.

89. $a=\sqrt{3}+1$

$b=\sqrt{3}-1$

$\frac{\sin A}{\sin B}=\frac{\sqrt{3}+1}{\sqrt{3}-1}=\frac{3+1+2\sqrt{3}}{2}=2+\sqrt{3}$

$\frac{\sin A}{\sin(120-A)}=\sqrt{3}+2$

$\frac{\sin A}{\sin 12\cos A-\cos 12\sin A}=\sqrt{3}+2$

$\frac{1}{\frac{\sqrt{3}}{2}\cot A+\frac{1}{2}}=\sqrt{3}+2$

$$\frac{\sqrt{3}\cot A+1}{2}=\frac{1}{\sqrt{3}+2}$$
$$=\frac{\sqrt{3}-2}{-1}$$
$$\frac{\sqrt{3}\cot A+1}{2}=-\sqrt{3}+2$$
$$\sqrt{3}\cot A=4-2\sqrt{3}-1$$
$$\sqrt{3}\cot A=3-2\sqrt{3}$$
$$\cot A=\sqrt{3}-2$$
$$-\cot A=2-\sqrt{3}=\tan 15$$
$$\therefore\ A=105^o$$
$$\therefore B=15^o$$

90. $y=x^{3/2}-2 \qquad \frac{dy}{dx}=\frac{3}{2}\sqrt{x}$

Slope of normal $=-\frac{2}{3\sqrt{x}}$

Let point is $\left(x_1, x_1^{3/2}-2\right)$

$\therefore$ Normal $y-\left(x_1^{3/2}-2\right)=\frac{-2}{3\sqrt{x_1}}\left(x-x_1\right)$

Now put (1, 7) and solve it.

$\Rightarrow x_1=\frac{1}{3}$

$\therefore P\Rightarrow\left(\frac{1}{3}, 7+\frac{1}{3\sqrt{3}}\right), A\Rightarrow(1, 7)$

$\therefore AD=\frac{1}{6}\sqrt{\frac{7}{3}}$