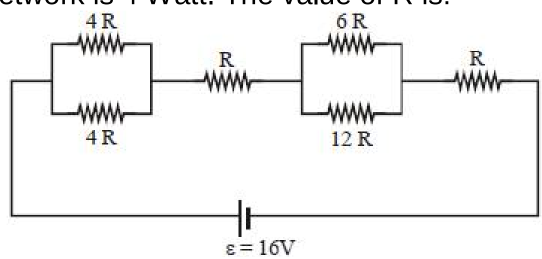
PART –A (PHYSICS)

1. The value of numerical aperature of the objective lens of a microscope is 1.25. If light of wavelength 5000 Å is used, the minimum separation between two points, to be seen as distinct, will be :

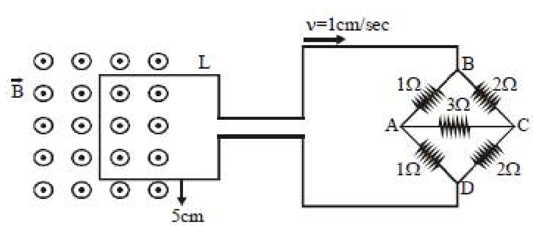
(A) 0.48 μm (B) 0.38 μm
(C) 0.24 μm (D) 0.12 μm

2. The resistive network shown below is connected to a D.C. source of 16 V. The power consumed by the network is 4 Watt. The value of R is:



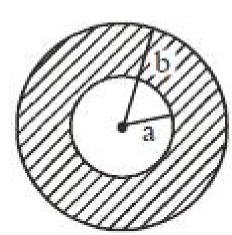
(A) 8 Ω (B) 6 Ω
(B) 16 Ω (D) 1 Ω

3. The figure shows a square loop L of side 5 cm which is connected to a network of resistances. The whole set up is moving towards right with a constant speed of 1 cms^{-1}. At some instant, a part of L is in a uniform magnetic field of 1 T, perpendicular to the plane of the loop. If the resistance of L is 1.7 Ω, the current in the loop at that instant will be close to :



(A) 115 μA (B) 170 μA
(C) 60 μA (D) 150 μA

4. A circular disc of radius b has a hole of radius a at its centre (see figure). If the mass per unit area of the disc varies as $\left(\frac{\sigma_0}{r}\right)$, then the radius of gyration of the disc about its axis passing through the centre is :



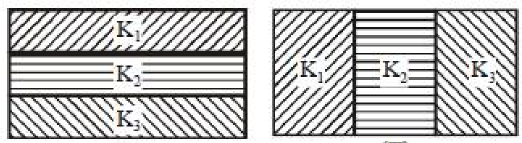
(A) $\frac{a+b}{3}$ (B) $\sqrt{\frac{a^2+b^2+ab}{3}}$

(C) $\frac{a+b}{2}$ (D) $\sqrt{\frac{a^2+b^2+ab}{2}}$

5. A man (mass = 50 kg) and his son (mass = 20 kg) are standing on a frictionless surface facing each other. The man pushes his son so that he starts moving at a speed of 0.70 ms^{-1} with respect to the man. The speed of the man with respect to the surface is :

(A) 0.47 ms^{-1} (B) 0.28 ms^{-1}
(C) 0.14 ms^{-1} (D) 0.20 ms^{-1}

6. Two identical parallel plate capacitors, of capacitance C each, have plates of area A, separated by a distance d. The space between the plates of the two capacitors, is filled with three dielectrics, of equal thickness and dielectric constants K_1, K_2 and K_3. The first capacitor is filled as shown in fig. I, and the second one is filled as shown in fig. II.
If these two modified capacitors are charged by the same potential V, the ratio of the energy stored in the two, would be (E_1 refers to capacitor (I) and E_2 to capacitor (II)) :



(I) (II)

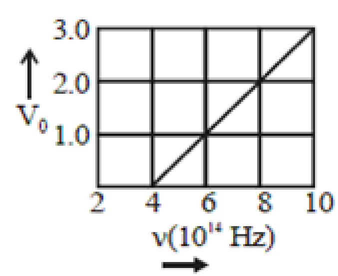
(A) $\frac{E_1}{E_2} = \frac{K_1K_2K_3}{(K_1+K_2+K_3)(K_2K_3+K_3K_1+K_1K_2)}$ (B) $\frac{E_1}{E_2} = \frac{9K_1K_2K_3}{(K_1+K_2+K_3)(K_2K_3+K_3K_1+K_1K_2)}$

(C) $\frac{E_1}{E_2} = \frac{(K_1+K_2+K_3)(K_2K_3+K_3K_1+K_1K_2)}{9K_1K_2K_3}$ (D) $\frac{E_1}{E_2} = \frac{(K_1+K_2+K_3)(K_2K_3+K_3K_1+K_1K_2)}{K_1K_2K_3}$

7. Two moles of helium gas is mixed with three moles of hydrogen molecules (taken to be rigid). What is the molar specific heat of mixture at constant volume? (R = 8.3 J/mol K)

(A) 17.4 J/mol K (B) 15.7 J/mol K
(C) 19.7 J/mol K (D) 21.6 J/mol K

8. The stopping potential V_0 (in volt) as a function of frequency (ν) for a sodium emitter, is shown in the figure. The work function of sodium, from the data plotted in the figure, will be :
(Given : Planck's constant (h) = 6.63×10^{-34} Js, electron charge e = 1.6×10^{-19} C)



(A) 1.82 eV (B) 1.66 eV
(C) 2.12 eV (D) 1.95 eV

9. At 40°C, a brass wire of 1 mm is hung from the ceiling. A small mass, M is hung from the free end of the wire. When the wire is cooled down from 40ºC to 20ºC it regains its original length of 0.2 m. The value of M is close to :
(Coefficient of linear expansion and Young's modulus of brass are 10^{-5}/ºC and 10^{11} N/m^2, respectively; g = 10 ms^{-2})

(A) 0.5 kg (B) 9 kg
(C) 0.9 kg (C) 1.5 kg

10. A magnetic compass needle oscillates 30 times per minute at a place where the dip is 45°, and 40 times per minute where the dip is 30º. If B_1 and B_2 are respectively the total magnetic field due to the earth at the two places, then the ratio B_1/B_2 is best given by :

(A) 3.6 (B) 1.8
(C) 2.2 (D) 0.7

11. An excited He^+ ion emits two photons in succession, with wavelengths 108.5 nm and 30.4 nm, in making a transition to ground state. The quantum number n, corresponding to its initial excited state is (for photon of wavelength λ, energy $E = \frac{1240 \text{ eV}}{\lambda(\text{in nm})}$:)

(A) n = 4
(B) n = 6
(C) n = 5
(D) n = 7

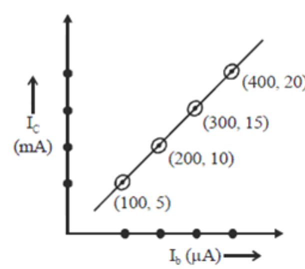
12. When M_1 gram of ice at –10ºC (specific heat = 0.5 cal $g^{-1}°C^{-1}$) is added to M_2 gram of water
at 50ºC, finally no ice is left and the water is at 0ºC. The value of latent heat of ice, in cal g^{-1} is:

(A) $\frac{50M_2}{M_1} - 5$
(B) $\frac{5M_2}{M_1} - 5$
(C) $\frac{50M_2}{M_1}$
(D) $\frac{5M_1}{M_2} - 50$

13. A shell is fired from a fixed artillery gun with an initial speed u such that it hits the target on the ground at a distance R from it. If t_1 and t_2 are the values of the time taken by it to hit the target in two possible ways, the product t_1t_2 is:

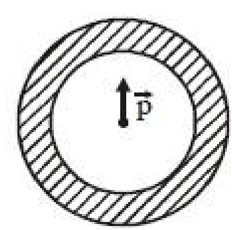
(A) 2R / g
(B) R / 4g
(C) R / g
(D) R/2g

14. The transfer characteristic curve of a transistor, having input and output resistance 100 Ω and 100 kΩ respectively, is shown in the figure. The Voltage and Power gain, are respectively :



(A) $5 \times 10^4, 2.5 \times 10^6$
(B) $5 \times 10^4, 5 \times 10^6$
(C) $5 \times 10^4, 5 \times 10^5$
(D) $2.5 \times 10^4, 2.5 \times 10^6$

15. Shown in the figure is a shell made of a conductor. It has inner radius a and outer radius b, and carries charge Q. At its centre is a dipole $\vec{P}$ as shown. In this case :



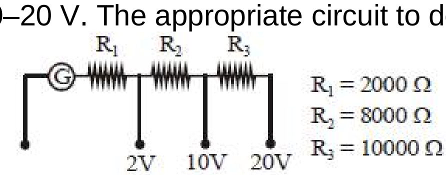
(A) Surface charge density on the inner surface of the shell is zero everywhere.
(B) Electric field outside the shell is the same as that of a point charge at the centre of the shell.
(C) Surface charge density on the inner surface is uniform and equal to $\frac{(Q/2)}{4\pi a^2}$
(D) Surface charge density on the outer surface depends on $|\vec{P}|$

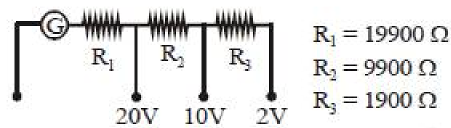
16. Which of the following combinations has the dimension of electrical resistance (ϵ_0 is the permittivity of vacuum and μ_0 is the permeability of vacuum)?

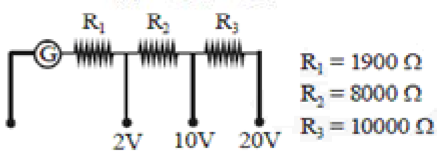
(A) $\sqrt{\frac{\epsilon_0}{\mu_0}}$ (B) $\frac{\mu_0}{\epsilon_0}$

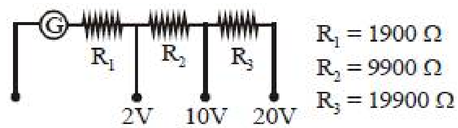
(C) $\frac{\epsilon_0}{\mu_0}$ (D) $\sqrt{\frac{\mu_0}{\epsilon_0}}$

17. A galvanometer of resistance 100 Ω has 50 divisions on its scale and has sensitivity of 20 μA / division. It is to be converted to a voltmeter with three ranges, of 0–2 V, 0–10 V and 0–20 V. The appropriate circuit to do so is :

(A) 

(B) 

(C) 

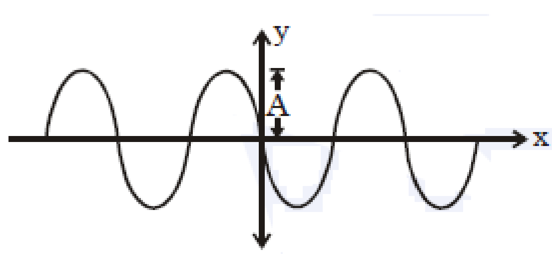
(D) 

18. In a double slit experiment, when a thin film of thickness t having refractive index μ is introduced in front of one of the slits, the maximum at the centre of the fringe pattern shifts by one fringe width. The value of t is (λ is the wavelength of the light used) :

(A) $\frac{\lambda}{2(\mu-1)}$ (B) $\frac{\lambda}{(\mu-1)}$

(C) $\frac{\lambda}{(2\mu-1)}$ (D) $\frac{2\lambda}{(\mu-1)}$

19. A progressive wave travelling along the positive x-direction is represented by y(x, t) = A sin
(kx – ωt + ϕ). Its snapshot at t = 0 is given in the figure:

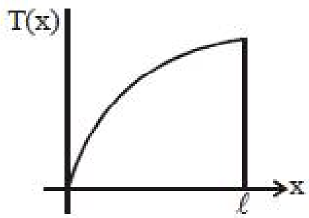


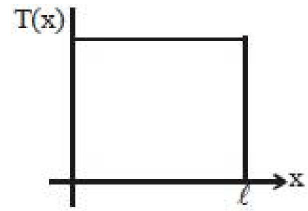
For this wave, the phase ϕ is :

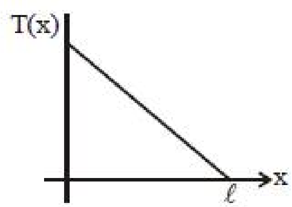
(A) π (B) $\frac{\pi}{2}$

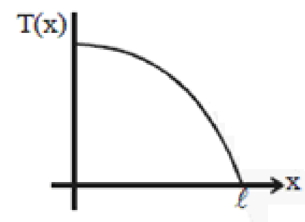
(C) $-\frac{\pi}{2}$ (D) 0

20. A uniform rod of length ℓ is being rotated in a horizontal plane with a constant angular speed
about an axis passing through one of its ends. If the tension generated in the rod due to rotation is T(x) at a distance x from the axis, then which of the following graphs depicts it most closely?

(A) 

(B) 

(C) 

(D) 

21. A point dipole $\vec{p} = -p_0\hat{x}$ is kept at the origin. The potential and electric field due to this dipole on the y-axis at a distance d are, respectively : (Take V = 0 at infinity)

(A) $\frac{|\vec{p}|}{4\pi\varepsilon_0 d^2}, \frac{-\vec{p}}{4\pi\varepsilon_0 d^3}$

(B) $0, \frac{\vec{p}}{4\pi\varepsilon_0 d^3}$

(C) $\frac{|\vec{p}|}{4\pi\varepsilon_0 d^2}, \frac{\vec{p}}{4\pi\varepsilon_0 d^3}$

(D) $0, \frac{-\vec{p}}{4\pi\varepsilon_0 d^3}$

22. An electromagnetic wave is represented by the electric field $\vec{E} = E_0\hat{n}\sin[\omega t + (6y - 8z)]$. Taking unit vectors in x, y and z directions to be $\hat{i}, \hat{j}, \hat{k}$, the direction of propogation $\hat{s}$, is :

(A) $\hat{S} = \left(\frac{-3\hat{j} + 4\hat{k}}{5}\right)$

(B) $\hat{S} = \left(\frac{4\hat{j} - 3\hat{k}}{5}\right)$

(C) $\hat{S} = \left(\frac{-4\hat{k} + 3\hat{j}}{5}\right)$

(D) $\hat{S} = \left(\frac{-3\hat{i} - 4\hat{j}}{5}\right)$

23. A submarine (A) travelling at 18 km/hr is being chased along the line of its velocity by another submarine (B) travelling at 27 km/hr. B sends a sonar signal of 500 Hz to detect A and receives a reflected sound of frequency ν. The value of ν is close to :
(Speed of sound in water = 1500 ms^{-1})

(A) 499 Hz
(B) 502 Hz
(C) 504 Hz
(D) 507 Hz

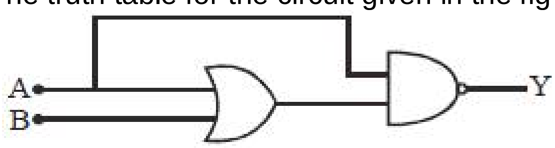
24. The trajectory of a projectile near the surface of the earth is given as $y = 2x - 9x^2$. If it were launched at an angle 0_0 with speed v_0 then ($g = 10\ ms^{-2}$) :

(A) $\theta_0 = \cos^{-1}\left(\frac{1}{\sqrt{5}}\right)$ and $v_0 = \frac{5}{3} ms^{-1}$
(B) $\theta_0 = \cos^{-1}\left(\frac{2}{\sqrt{5}}\right)$ and $v_0 = \frac{3}{5} ms^{-1}$
(C) $\theta_0 = \sin^{-1}\left(\frac{2}{\sqrt{5}}\right)$ and $v_0 = \frac{3}{5} ms^{-1}$
(D) $\theta_0 = \sin^{-1}\left(\frac{1}{\sqrt{5}}\right)$ and $v_0 = \frac{5}{3} ms^{-1}$

25. A thin ring of 10 cm radius carries a uniformly distributed charge. The ring rotates at a constant angular speed of $40\ \pi$ rad s^{-1} about its axis, perpendicular to its plane. If the magnetic field at its centre is 3.8×10^{-9} T, then the charge carried by the ring is close to ($\mu_0 = 4\pi \times 10^{-7} N / A^2$).

(A) 2×10^{-6} C
(B) 7×10^{-6} C
(C) 4×10^{-5} C
(D) 3×10^{-5} C

26. The truth table for the circuit given in the fig is :



(A)

A	B	Y
0	0	1
0	1	0
1	0	0
1	1	0

(B)

A	B	Y
0	0	0
0	1	0
1	0	1
1	1	1

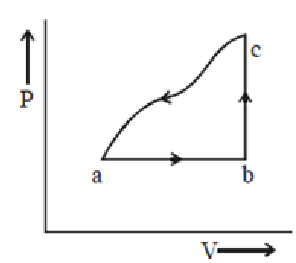
(C)

A	B	Y
0	0	1
0	1	1
1	0	1
1	1	1

(D)

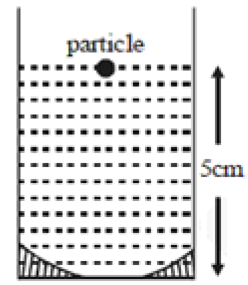
A	B	Y
0	0	1
0	1	1
1	0	0
1	1	0

27. A sample of an ideal gas is taken through the cyclic process abca as shown in the figure. The change in the internal energy of the gas along the path ca is –180 J. The gas absorbs 250 J of heat along the path ab and 60 J along the path bc. The work done by the gas along the path abc is :



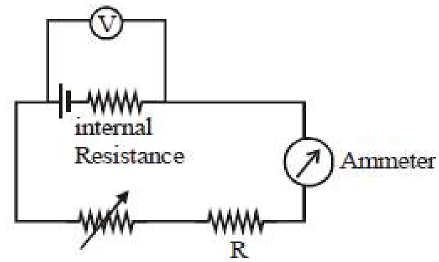
(A) 120 J
(B) 100 J
(C) 140 J
(D) 130 J

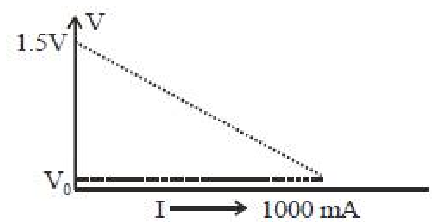
28. A concave mirror has radius of curvature of 40 cm. It is at the bottom of a glass that has water filled up to 5 cm (see figure). If a small particle is floating on the surface of water, its image as seen, from directly above the glass, is at a distance d from the surface of water. The value of d is close to : (Refractive index of water = 1.33)



(A) 13.4 cm
(B) 8.8 cm
(C) 6.7 cm
(D) 11.7 cm

29. To verify Ohm's law, a student connects the voltmeter across the battery as, shown in the figure. The measured voltage is plotted as a function of the current, and the following graph is obtained:





If V_0 is almost zero, identify the correct statement:
(A) The potential difference across the battery is 1.5 V when it sends a current of 1000 mA.
(B) The emf of the battery is 1.5 V and the value of R is 1.5 Ω
(C) The emf of the battery is 1.5 V and its internal resistance is 1.5 Ω
(D) The value of the resistance R is 1.5 Ω

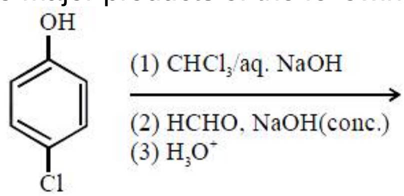
30. A person of mass M is, sitting on a swing of length L and swinging with an angular amplitude θ_0. If the person stands up when the swing passes through its lowest point, the work done by him, assuming that his centre of mass moves by a distance ℓ ($\ell << L$), is close to :
(A) $Mg\ell(1+\theta_0^2)$
(B) $Mg\ell(1-\theta_0^2)$
(C) $Mg\ell$
(D) $Mg\ell\left(1+\frac{\theta_0^2}{2}\right)$

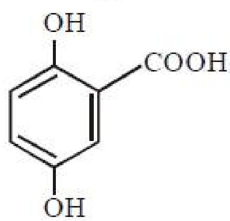
PART –B (CHEMISTRY)

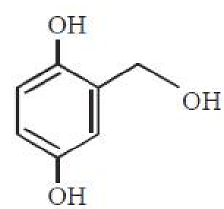
31. The mole fraction of a solvent in aqueous solution of a solute is 0.8.The molality (in mol kg^{-1}) of the aqueous solution is

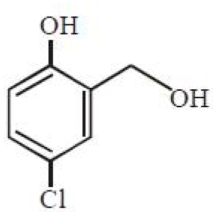
(A) 13.88×10^{-2} (B) 13.88×10^{-1}
(C) 13.88 (D) 13.88×10^{-3}

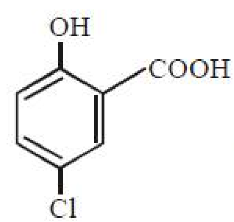
32. The major products of the following reaction are:



(A)  and Methanol

(B)  and Formic acid

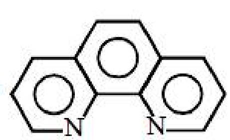
(C)  and Formic acid

(D)  and Methanol

33. The correct statement among the following is

(A) $(SiH_3)_3N$ is planar and less basic than $(CH_3)_3N$.
(B) $(SiH_3)_3N$ is planar and more basic than $(CH_3)_3N$.
(C) $(SiH_3)_3N$ is pyramidal and less basic than $(CH_3)_3N$.
(D) $(SiH_3)_3N$ is pyramidal and more basic than $(CH_3)_3N$.

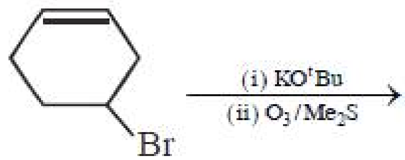
34. The complex ion that will lose its crystal field stabilization energy upon oxidation of its metal
to +3 state is

(Phen =  and ignore pairing energy)

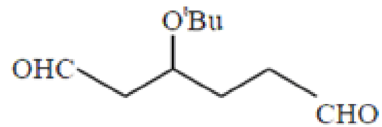
(A) $[Ni(phen)_3]^{2+}$ (B) $[Zn(phen)_3]^{2+}$
(C) $[Co(phen)_3]^{2+}$ (D) $[Fe(phen)_3]^{2+}$

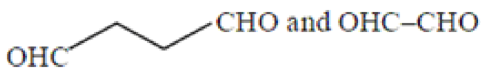
35. The correct set of species responsible for the photochemical smog is :
(A) NO, NO_2, O_3 and hydrocarbons
(B) CO_2, NO_2 SO_2 and hydrocarbons
(C) N_2, NO_2 and hydrocarbons
(D) N_2, O_2, O_3 and hydrocarbons

36. The major product(s) obtained in the following reaction is/ are:



(A) 

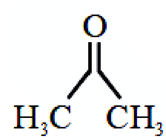
(B) 

(C) 

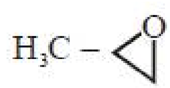
(D) 

37. The major product of the following addition reaction is:
$H_3C-CH=CH_2 \xrightarrow{Cl_2/H_2O}$

(A) $\underset{\displaystyle Cl}{CH_3-\underset{|}{CH}}-\underset{\underset{\displaystyle OH}{|}}{CH_2}$

(B) 

(C) $\underset{\displaystyle OH}{H_3C-\underset{|}{CH}}-\underset{\underset{\displaystyle Cl}{|}}{CH_2}$

(D) 

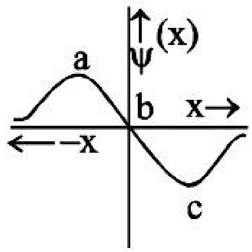
38. The correct sequence of thermal stability of the following carbonates is :
(A) $BaCO_3 < SrCO_3 < CaCO_3 < MgCO_3$
(B) $BaCO_3 < CaCO_3 < SrCO_3 < MgCO_3$
(C) $MgCO_3 < CaCO_3 < SrCO_3 < BaCO_3$
(D) $MgCO_3 < SrCO_3 < CaCO_3 < BaCO_3$

39. The group number, number of valence electrons and valency of an element with atomic number 15, respectively, are:
(A) 15, 5 and 3
(B) 15, 6 and 2
(C) 16, 5 and 2
(D) 16, 6 and 3

40. Peptization is a :
(A) process of converting soluble particles to form colloidal solution
(B) process of converting precipitate into colloidal solution
(C) process of converting a colloidal solution into precipitate
(D) process of bringing colloidal molecule into solution

41. The metal that gives hydrogen gas upon treatment with both acid as well as base is:
(A) iron
(B) magnesium
(C) zinc
(D) mercury

42. The electrons are more likely to be found:



(A) in the region a and b
(B) in the region a and c
(C) only in the region a
(D) only in the region c

43. An example of a disproportionation reaction is :

(A) $2KMnO_4 \longrightarrow K_2MnO_4 + MnO_2 + O_2$

(B) $2NaBr + Cl_2 \longrightarrow 2naCl + Br_2$

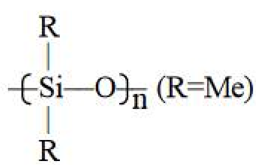
(C) $2CuBr \longrightarrow CuBr_2 + Cu$

(D) $2MnO_4^- + 10I^- + 16H^+ \longrightarrow 2Mn^{2+} + 5I_2 + 8H_2O$

44. 5 moles of AB_2 weigh 125×10^{-3} kg and 10 moles of A_2B_2 weigh 300×10^{-3} kg. The molar mass of $A(M_A)$ and molar mass of $B(M_B)$ in kg mol^{-1} are :

(A) $M_A = 50 \times 10^{-3}$ and $M_B = 25 \times 10^{-3}$
(B) $M_A = 10 \times 10^{-3}$ and $M_B = 5 \times 10^{-3}$
(C) $M_A = 5 \times 10^{-3}$ and $M_B = 10 \times 10^{-3}$
(D) $M_A = 25 \times 10^{-3}$ and $M_B = 50 \times 10^{-3}$

45. The basic structural unit of feldspar, zeolites, mica and asbestos is :

(A) 

(B) $(SiO_3)^{2-}$

(C) SiO_2

(D) $(SiO_4)^{4-}$

46. Enthalpy of sublimation of iodine is 24 cal g^{-1} at 200°C. If specific heat of $I_2(s)$ and I_2 (vap) are 0.055 and 0.031 cal $g^{-1}K^{-1}$ respectively, then enthalpy of sublimation of iodine at 250°C in cal g^{-1} is :

(A) 2.85
(B) 11.4
(C) 5.7
(D) 22.8

47. Which of the following is a thermosetting polymer?

(A) PVC
(B) Bakelite
(C) Buna-N
(D) Nylon 6

48. An element has a face-centred cubic (fcc) structure with a cell edge of a. The distance between the centres of two nearest tetrahedral voids in the lattice is:

(A) $\frac{a}{2}$
(B) $\sqrt{2}a$
(C) $\frac{3}{2}a$
(D) a

49. An organic compound 'A' is oxidized with Na_2O_2 followed by boiling with HNO_3.The resultant solution is then treated with ammonium molybdate to yield a yellow precipitate. Based on above observation, the element present in the given compound is :

(A) Phosphorus (B) Sulphur

(C) Nitrogen (D) Fluorine

50. The idea of froth floatation method came from a person X and this method is related to the process Y of ores. X and Y, respectively, are:

(A) washer woman and concentration (B) fisher woman and concentration

(C) fisher man and reduction (D) washer man and reduction

51. In the following reaction; $xA \longrightarrow yB$

$$\log_{10}\left[-\frac{d[A]}{dt}\right] = \log_{10}\left[\frac{d[B]}{dt}\right] + 0.3010$$

'A' and 'B' respectively can be :

(A) C_2H_2 and C_6H_6 (B) n-Butane and Iso-butane

(C) N_2O_4 and NO_2 (D) C_2H_4 and C_4H_8

52. Glucose and Galactose are having identical configuration in all the positions except position.

(A) C–4 (B) C–3

(C) C–5 (D) C–2

53. Which of the following statements is not true about RNA?

(A) It has always double stranded α-helix structure

(B) It is present in the nucleus of the cell

(C) It controls the synthesis of protein

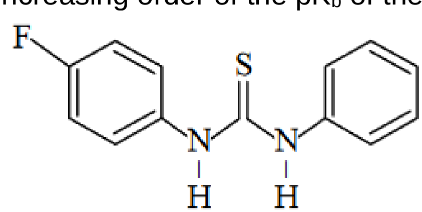
(D) It usually does not replicate

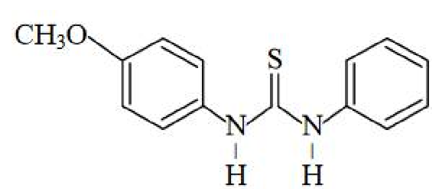
54. What is the molar solubility of $Al(OH)_3$ in 0.2 M NaOH solution? Given that, solubility product of $Al(OH)_3 = 2.4 \times 10^{-24}$:

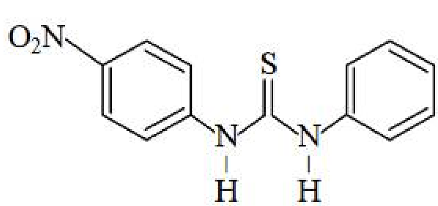
(A) 3×10^{-19} (B) 12×10^{-21}

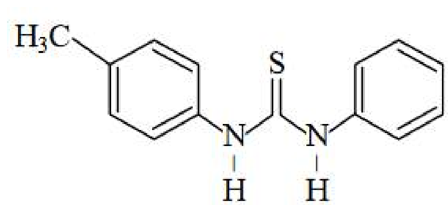
(C) 12×10^{-23} (D) 3×10^{-22}

55. The increasing order of the pK_b of the following compound is :

(A) 

(B) 

(C) 

(D) 

(A) (c) < (a) < (d) < (b) (B) (b) < (d) < (a) < (c)

(C) (a) < (c) < (d) < (b) (D) (b) < (d) < (c) < (a)

56. Given :

$Co^{3+}e^{-} \longrightarrow Co^{2+}$; $E^{\circ} = 1.81V$

$Pb^{4} + 2e^{-} \longrightarrow Pb^{2+}$; $E^{\circ} = +1.67\,V$

$Ce^{4+} + e^{-} \longrightarrow Ce^{3+}$; $E^{\circ} + 1.61V$

$Bi^{3+} + 3e^{-} \longrightarrow Bi$; $E^{\circ} = +0.20V$

Oxidizing power of the species will increase in the order

(A) $Ce^{4+} < Pb^{4+} < Bi^{3+} < Co^{3+}$
(B) $Co^{3+} < Pb^{4+} < Ce^{4+} < Bi^{3+}$
(C) $Bi^{3+} < Ce^{4+} < Pb^{4+} < Co^{3+}$
(D) $Co^{3+} < Ce^{4+} < Bi^{3+} < Pb^{4+}$

57. But-2-ene on reaction with alkaline $KMnO_4$ at elevated temperature followed by acidification will give :

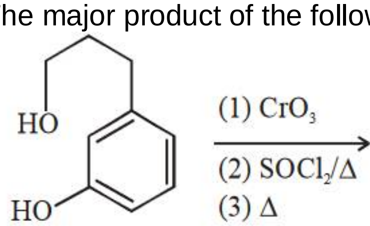
(A) One molecule of CH_3CHO and one molecule of CH_3COOH

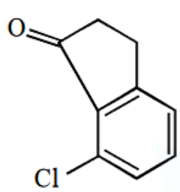
(B) 2 molecules of CH_3CHO

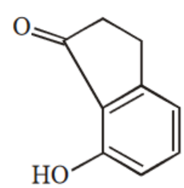
(C) 2 molecules of CH_3COOH

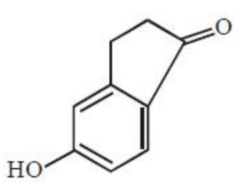
(D) $CH_3-\underset{\displaystyle OH}{\underset{|}{CH}}-\underset{\displaystyle OH}{\underset{|}{CH}}-CH_3$

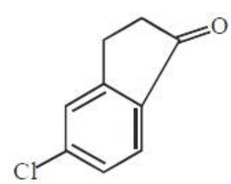
58. The major product of the following reaction is :



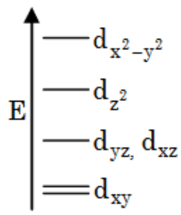
(A) 

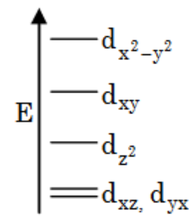
(B) 

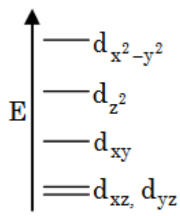
(C) 

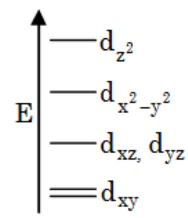
(D) 

59. Complete removal of both the axial ligands (along the z-axis) from an octahedral complex leads to which of the following splitting patterns? (relative orbital energies not on scale).

(A) 

(B) 

(C) 

(D) 

60. An ideal gas is allowed to expand from 1 L to 10 L against a constant external pressure of 1bar. The work done in kJ is:

(A) – 9.0
(B) – 0.9
(C) – 2.0
(D) + 10.0

PART–C (MATHEMATICS)

61. Let P be the point of intersection of the common tangents to the parabola $y^2 = 12x$ and the hyperbola $8x^2 - y^2 = 8$. If S and S' denote the foci of the hyperbola where S lies on the positive x-axis then P divides SS' in a ratio:

(A) 2:1 (B) 13:11
(C) 5:4 (D) 14:13

62. If three of the six vertices of a regular hexagon are chosen at random, then the probability that the triangle formed with these chosen vertices is equilateral is :

(A) $\frac{3}{10}$ (B) $\frac{1}{5}$
(C) $\frac{1}{10}$ (D) $\frac{3}{20}$

63. Let $f : R \to R$ be a continuously differentiable function such that $f(2) = 6$ and $f'(2) = \frac{1}{48}$. If $\int_6^{f(x)} 4t^3 dt = (x-2)g(x)$, then $\lim_{x\to 2} g(x)$ is equal to :

(A) 24 (B) 18
(C) 12 (D) 36

64. The integral $\int \frac{2x^3 - 1}{x^4 + x} dx$ is equal to :
(Here C is a constant of integration)

(A) $\frac{1}{2}\log_e \frac{|x^3+1|}{x^2} + C$ (B) $\frac{1}{2}\log_e \frac{|x^3+1|^2}{|x^3|} + C$
(C) $\log_e \left|\frac{|x^3+1|}{x}\right| + C$ (D) $\log_e \frac{|x^3+1|}{x^2} + C$

65. The coefficient of x^{18} in the product $(1+x)(1-x)^{10}(1+x+x^2)^9$ is :

(A) 84 (B) 126
(C) –126 (D) –84

66. The number of ways of choosing 10 objects out of 31 objects of which 10 are identical and the remaining 21 are distinct, is :

(A) 2^{20} (B) $2^{20} + 1$
(C) 2^{21} (D) $2^{20} - 1$

67. Let a random variable X have a binomial distribution with mean 8 and variance 4. If $P(X \le 2) = \frac{k}{2^{16}}$, then k is equal to :

(A) 17 (B) 137
(C) 1 (D) 121

68. The equation $|z-i| = |z-1|, i = \sqrt{-1}$, represents :

(A) a circle of radius $\frac{1}{2}$

(B) the line through the origin with slope 1

(C) a circle of radius 1

(D) the line through the origin with slope –1

69. If m is the minimum value of k for which the function $f(x) = x\sqrt{kx - x^2}$ is increasing in the interval [0,3] and M is the maximum value of f in [0, 3] when k = m, then the ordered pair (m, M) is equal to :

(A) $(5, 3\sqrt{6})$

(B) $(4, 3\sqrt{2})$

(C) $(3, 3\sqrt{3})$

(D) $(4, 3\sqrt{3})$

70. If the data $x_1, x_2, ..., x_{10}$ is such that the mean of first four of these is 11, the mean of the remaining six is 16 and the sum of squares of all of these is 2,000; then the standard deviation of this data is :

(A) $2\sqrt{2}$

(B) 2

(C) 4

(D) $\sqrt{2}$

71. If the area (in sq. units) of the region $\{(x, y) : y^2 \le 4x, x + y \le 1, x \ge 0, y \ge 0\}$ is $a\sqrt{2} + b$, then a – b is equal to :

(A) $\frac{10}{3}$

(B) 6

(C) $\frac{8}{3}$

(D) $-\frac{2}{3}$

72. Consider the differential equation, $y^2 dx + \left(x - \frac{1}{y}\right) dy = 0$. If value of y is 1 when x = 1, then the value of x for which y = 2, is :

(A) $\frac{3}{2} - \sqrt{e}$

(B) $\frac{1}{2} + \frac{1}{\sqrt{e}}$

(C) $\frac{3}{2} - \frac{1}{\sqrt{e}}$

(D) $\frac{5}{2} + \frac{1}{\sqrt{e}}$

73. If α and β are the roots of the equation $375x^2 – 25x – 2 = 0$, then

$\lim_{n\to\infty} \sum_{r=1}^{n} \alpha^r + \lim_{n\to\infty} \sum_{r=1}^{n} \beta^r$ is equal to :

(A) $\frac{1}{12}$

(B) $\frac{29}{358}$

(C) $\frac{7}{116}$

(D) $\frac{21}{346}$

74. A 2 m ladder leans against a vertical wall. If the top of the ladder begins to slide down the wall at the rate 25 cm/ sec., then the rate (in cm/sec.) at which the bottom of the ladder slides away from the wall on the horizontal ground when the top of the ladder is 1 m above the ground is :

(A) 25 (B) $\frac{25}{3}$

(C) $25\sqrt{3}$ (D) $\frac{25}{\sqrt{3}}$

75. The number of solutions of the equation $1 + \sin^4 x = \cos^2 3x$, $x \in \left[-\frac{5\pi}{2}, \frac{5\pi}{2}\right]$ is :

(A) 3 (B) 4
(C) 5 (D) 7

76. If $e^y + xy = e$, the ordered pair $\left(\frac{dy}{dx}, \frac{d^2y}{dx^2}\right)$ at x = 0 is equal to :

(A) $\left(\frac{1}{e}, -\frac{1}{e^2}\right)$ (B) $\left(\frac{1}{e}, \frac{1}{e^2}\right)$

(C) $\left(-\frac{1}{e}, \frac{1}{e^2}\right)$ (D) $\left(-\frac{1}{e}, -\frac{1}{e^2}\right)$

77. The value of $\sin^{-1}\left(\frac{12}{13}\right) - \sin^{-1}\left(\frac{3}{5}\right)$ is equal to :

(A) $\pi - \cos^{-1}\left(\frac{33}{65}\right)$ (B) $\pi - \sin^{-1}\left(\frac{63}{65}\right)$

(C) $\frac{\pi}{2} - \cos^{-1}\left(\frac{9}{65}\right)$ (D) $\frac{\pi}{2} - \sin^{-1}\left(\frac{56}{65}\right)$

78. If the normal to the ellipse $3x^2 + 4y^2 = 12$ at a point P on it is parallel to the line, $2x + y = 4$ and the tangent to the ellipse at P passes through Q(4, 4) then PQ is equal to :

(A) $\frac{\sqrt{157}}{2}$ (B) $\frac{5\sqrt{5}}{2}$

(C) $\frac{\sqrt{221}}{2}$ (D) $\frac{\sqrt{61}}{2}$

79. Let $\vec{a} = 3\hat{i} + 2\hat{j} + 2\hat{k}$ and $\vec{b} = \hat{i} + 2\hat{j} - 2\hat{k}$ be two vectors. If a vector perpendicular to both the vectors $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$ has the magnitude 12 then one such vector is:

(A) $4\left(2\hat{i} - 2\hat{j} - \hat{k}\right)$ (B) $4\left(2\hat{i} - 2\hat{j} + \hat{k}\right)$

(C) $4\left(2\hat{i} + 2\hat{j} + \hat{k}\right)$ (D) $4\left(2\hat{i} + 2\hat{j} - \hat{k}\right)$

80. The equation $y = \sin x \sin(x + 2) - \sin^2(x+1)$ represents a straight line lying in :

(A) first, third and fourth quadrants (B) first, second and fourth quadrants
(C) third and fourth quadrants only (D) second and third quadrants only

81. For $x \in \left(0, \frac{3}{2}\right)$, let $f(x) = \sqrt{x}$, $g(x) = \tan x$ and $h(x) = \frac{1-x^2}{1+x^2}$. If $\phi(x) = ((hof)og)(x)$, then $\phi\left(\frac{\pi}{3}\right)$ is equal to :

(A) $\tan\frac{11\pi}{12}$
(B) $\tan\frac{\pi}{12}$
(C) $\tan\frac{5\pi}{12}$
(D) $\tan\frac{7\pi}{12}$

82. If the angle of intersection at a point where the two circles with radii 5 cm and 12 cm intersect is 90°, then the length (in cm) of their common chord is :

(A) $\frac{13}{2}$
(B) $\frac{120}{13}$
(C) $\frac{13}{5}$
(D) $\frac{60}{13}$

83. If $\int_0^{\frac{\pi}{2}} \frac{\cot x}{\cot x + \operatorname{cosec} x} dx = m(\pi + n)$, then $m \cdot n$ is equal to

(A) 1
(B) $\frac{1}{2}$
(C) $-\frac{1}{2}$
(D) –1

84. Let S_n denote the sum of the first n terms of an A.P.. If $S_4 = 16$ and $S_6 = -48$, then S_{10} is equal to :

(A) –410
(B) –260
(C) –320
(D) –380

85. If B = $\begin{bmatrix} 5 & 2\alpha & 1 \\ 0 & 2 & 1 \\ \alpha & 3 & -1 \end{bmatrix}$ is the inverse of a 3×3 matrix A, then the sum of all values of α for which det (A) + 1 = 0, is :

(A) 0
(B) –1
(C) 1
(D) 2

86. If the line $\frac{x-2}{3} = \frac{y+1}{2} = \frac{z-1}{1}$ intersects the plane 2x + 3y – z + 13 = 0 at a point P and the plane 3x + y + 4z = 16 at a point Q, then PQ is equal to :

(A) $2\sqrt{14}$
(B) 14
(C) $2\sqrt{7}$
(D) $\sqrt{14}$

87. If the volume of parallelepiped formed by the vectors $\hat{i} + \lambda\hat{j} + \hat{k}$, $\hat{j} + \lambda\hat{k}$ and $\lambda\hat{i} + \hat{k}$ is minimum, then λ is equal to :

(A) $\sqrt{3}$
(B) $\frac{1}{\sqrt{3}}$
(C) $-\frac{1}{\sqrt{3}}$
(D) $-\sqrt{3}$

88. If A is a symmetric matrix and B is a skew-symmetrix matrix such that A + B = $\begin{bmatrix} 2 & 3 \\ 5 & -1 \end{bmatrix}$, then AB is equal to :

(A) $\begin{bmatrix} 4 & -2 \\ 1 & -4 \end{bmatrix}$
(B) $\begin{bmatrix} 4 & -2 \\ -1 & -4 \end{bmatrix}$
(C) $\begin{bmatrix} -4 & 2 \\ 1 & 4 \end{bmatrix}$
(D) $\begin{bmatrix} -4 & -2 \\ -1 & 4 \end{bmatrix}$

89. If the truth value of the statement $p \to (\sim q \vee r)$ is false (F), then the truth values of the statement p, q, r are respectively :

(A) T, T, F
(B) F, T, T
(C) T, F, T
(D) T, F, F

90. For $x \in R$, let $[x]$ denote the greatest integer $\leq x$, then the sum of the series

$$\left[-\frac{1}{3}\right] + \left[-\frac{1}{3} - \frac{1}{100}\right] + \left[-\frac{1}{3} - \frac{2}{100}\right] + \cdots + \left[-\frac{1}{3} - \frac{99}{100}\right]$$

(A) –135
(B) –153
(C) –133
(D) –131

HINTS AND SOLUTIONS

PART A – PHYSICS

1. Numerical aperature of the microscope is given as

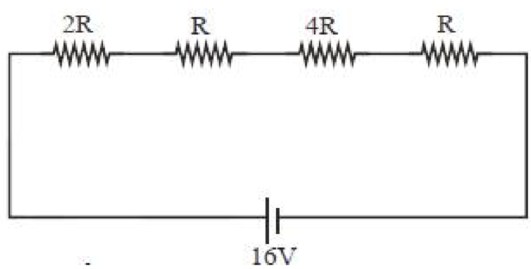
$$NA = \frac{0.61\lambda}{d}$$

Where d = minimum sparaton between two points to be seen as distinct

$$d = \frac{0.61\lambda}{NA} = \frac{(0.61)\times(5000\times10\ m^{-10})}{1.25} = 2.4 \times 10^{-7}\ m$$

$$= 0.24\ \mu m$$

2.



$$P = \frac{16^2}{8R} = 4$$

$$\therefore\quad R = 8\Omega$$

3. Since it is a balanced wheatstone bridge, its equivalent resistance = $\frac{4}{3}\Omega$

$\varepsilon = B\ell v = 5 \times 10^{-4}$ V

So total resistance

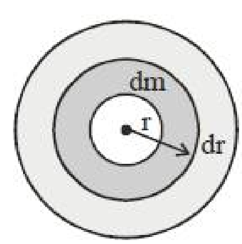
$$R = \frac{4}{3} + 1.7 \approx 3\Omega$$

$$\therefore\quad i = \frac{\varepsilon}{R} \approx 166\ \mu A \approx 170\ \mu A$$

4. $dI = (dm)r^2$

$= (\sigma\ dA)r^2$

$= \left(\frac{\sigma_0}{r}2\pi dr\right)r^2$ = ($\sigma_0\ 2\pi 0r^2dr$

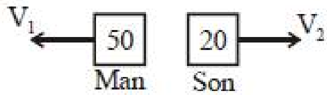


$$I = \int DI = \int_a^b \sigma_0 2\pi r^2 dr$$

$$= \sigma_0 2\pi\left(\frac{b^3 - a^3}{3}\right)$$

$$m = \int dm = \int \sigma dA$$

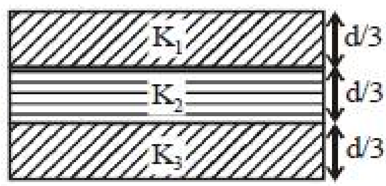
$$= \sigma_0 2\pi \int_a^b dr$$

5. 

$\Rightarrow$ $0 = 50V_1 - 20V_2$ and $V_1 + V_2 = 0.7$

$\Rightarrow$ $V_1 = 0.2$

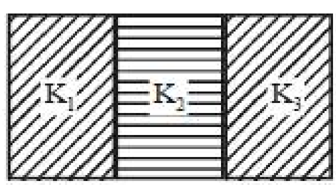
6.

I. 

$$C_1 = \frac{3\varepsilon_0 AK_1}{d}$$
$$C_2 = \frac{3\varepsilon_0 AK_2}{d}$$
$$C_3 = \frac{3\varepsilon_0 AK_3}{d}$$

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

$$\Rightarrow \quad C_{eq} = \frac{3\varepsilon_0 AK_1K_2K_3}{d(K_1K_2 + K_2K_3 + K_3K_1)} \qquad \ldots(i)$$

II. 

$$C_1 = \frac{\varepsilon_0 K_1 A}{3d}$$
$$C_2 = \frac{\varepsilon_0 K_2 A}{3d}$$
$$C_3 = \frac{\varepsilon_0 K_3 A}{3d}$$

$$C'_{eq} = C_1 + C_2 + C_3$$

$$= \frac{\varepsilon_0 A}{3d}(K_1 + K_2 + K_3) \qquad \ldots(ii)$$

Now,

$$\frac{E_1}{E_2} = \frac{\frac{1}{2}C_{eq} \cdot V^2}{\frac{1}{2}C'_{eq}V^2} = \frac{9K_1K_2K_3}{(K_1 + K_2 + K_3)(K_1K_2 + K_2K_3 + K_3K_1)}$$

7. $$f_{mix} = \frac{n_1f_1 + n_2f_2}{n_1 + n_2} = \frac{2\times3 + 3\times5}{5} = \frac{21}{5}$$

$$C_v = \frac{fR}{5} = \frac{21}{5}\times\frac{R}{2} = 17.4 \text{ J/mol K}$$

8. $h\nu = \phi + ev_0$

$$v_0 = \frac{h\nu}{e} - \frac{\phi}{e}$$

v_0 is zero for $\nu = 4 \times 10^{14}$ Hz

$$0 = \frac{hv}{e} - \frac{\phi}{e}$$

$$\Rightarrow \ \phi = h\nu$$

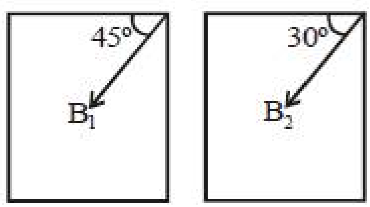
$$= \frac{6.63 \times 10^{-34} \times 4 \times 10^{14}}{1.6 \times 10^{-19}} = 1.66 \text{ eV}$$

9. $$Mg = \left(\frac{Ay}{\ell}\right)\Delta\ell$$

$Mg = (Ay)\alpha\Delta T = 2\pi$

It is closest to 9.

10.



$$f_1 = \frac{1}{2\pi}\sqrt{\frac{\mu B_1 \cos 45^o}{I}} \quad f_2 = \frac{1}{2\pi}\sqrt{\frac{\mu B_2 \cos 30^o}{I}}$$

$$\frac{f_1}{f_2} = \frac{B_1 \cos 45^o}{B_2 \cos 30^o} \qquad \therefore \ \frac{B_1}{B_2} \times 0.7$$

11. $$\frac{1}{\lambda} = R\left(\frac{1}{m^2} - \frac{1}{n^2}\right)z^2$$

$$\frac{1}{1085} = R\left(\frac{1}{m^2} - \frac{1}{n^2}\right)2^2$$

$\therefore$ m = 2

$\therefore$ n = 5

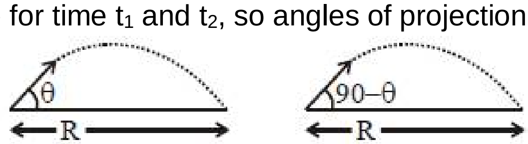
12. Heat lost = Heat gain

$\Rightarrow$ $M_2 \times 1 \times 50 = M_1 \times 0.5 \times 10 + M_1.L_f$

$$\Rightarrow \quad L_f = \frac{50M_2 - 5M_1}{M_1}$$

$$= \frac{50M_2}{M_1} - 5$$

13. Range will be same for time t_1 and t_2, so angles of projection will be 'θ' & '90° – θ'



$$t_1 = \frac{2u \sin\theta}{g} \ t_2 = \frac{2u\sin(90^o - \theta)}{g} \text{ and } R = \frac{u^2 \sin 2\theta}{g}$$

$$t_1 t_2 = \frac{4u^2 \sin\theta\cos\theta}{g^2} = \frac{2}{g}\left[\frac{2u^2\sin\theta\cos\theta}{g}\right]$$

$$= \frac{2R}{g}$$

14. $V_{gain} = \left(\frac{\Delta I_C}{\Delta I_B}\right)\frac{R_{out}}{R_{in}} = \left(\frac{5\times10^{-3}}{100\times10^{-6}}\right)\times10^{3}$

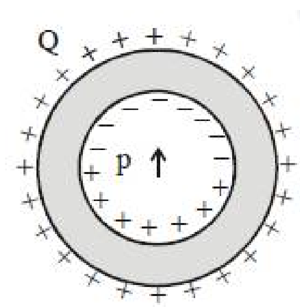
$= \frac{1}{20}\times10^{8} = 5\times10^{4}$

$P_{gain} = \left(\frac{\Delta I_C}{\Delta I_b}\right)(V_{gain})$

$= \left(\frac{5\times10^{-3}}{100\times10^{-6}}\right)(5\times10^{4})$

$= 2.5 \times 10^{6}$

15. Total charge of dipole = 0, so charge induced on outside surface = 0.
But due to non uniform electric field of dipole, the charge induced on inner surface is non zero and non uniform.
So, for any observer outside the shell, the resultant electric field is due to Q uniformly distributed on outer surface only and it is equal to.



$\boxed{E = \frac{KQ}{r^2}}$

16. $[\varepsilon_0] = M^{-1} L^{-3} T^{4} A^{2}$

$[\mu_0] = M L T^{-2} A^{-2}$

$[R] = M L^{2} T^{-3} A^{-2}$

$[R] = \left[\sqrt{\frac{\mu_0}{\varepsilon_0}}\right]$

17. $20 \times 50 \times 10^{-6} = 10^{-3}$ Amp.

$V_1 = \frac{2}{10^{-3}} = 100 + R_1$

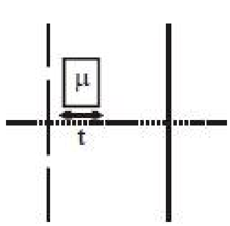
$1900 = R_1$

$V_2 = \frac{10}{10^{-3}} = (2000 + R_2)$

$R_2 = 8000$

$V_3 = \frac{20}{10^{-3}} = 10\times10^{3} + R_3 = 10 \times 10^{3}\ R_3$

18.

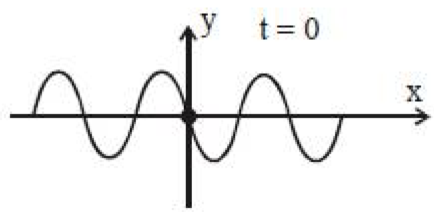


$\Delta x = (\mu - 1)t = 1\lambda$

for one maximum shift

$$t = \frac{\lambda}{\mu - 1}$$

19.

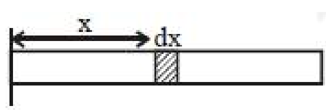


y = A sin (kx – wt + ϕ)

at x = 0, t = 0 and slope is negative

$\Rightarrow \ \phi = \pi$

20.



$$T = \int_{x=x}^{x=\ell} dm\omega^2 x = \int_{x=x}^{x=\ell} \frac{m}{\ell} dx\omega^2 x \ T$$

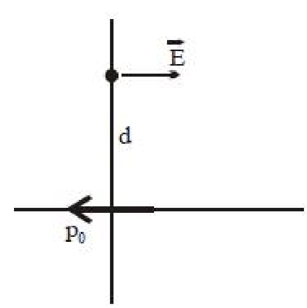
$$= \frac{m\omega^2}{2\ell}(\ell^2 - x^2)$$

$$T = \frac{m\omega^2}{2\ell}(\ell^2 - x^2)$$

21. V = 0

$$E = -\frac{K\vec{P}}{r^3}$$

$$= -\frac{\vec{p}}{4\pi\varepsilon_0 d^3}$$



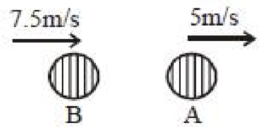
22. $\vec{E} = E_0\hat{n}\sin(\omega t + (6y - 8z)) = E_0\hat{n}\sin(\omega t + \vec{k}\cdot\vec{r})$

where $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ and $\vec{k}\cdot\vec{r} = 6y - 8z$

$\Rightarrow \ \vec{k} = 6\hat{j} - 8\hat{k}$

direction of propagation $\hat{s} = -\hat{k}$

$$= \left(\frac{-3\hat{j} + 4\hat{k}}{5}\right)$$

23.


f_0 = 500 Hz frequency received by B again =

$\longleftarrow$ 1500

(B) (A) & $\Rightarrow$

7.5 m/s $\longrightarrow$ $\longrightarrow$ 5 m/sec

$$f_2 = \left(\frac{1500+7.5}{1500+5}\right) \times \left(\frac{1500-5}{1500-7.5}\right) f_0 = 502 \text{ Hz}$$

24. Equation of trajectory is given as

$y = 2x - 9x^2$ …(A)

Comparing with equation:

$y = x \tan\theta - \dfrac{g}{2u^2 \cos^2\theta} \cdot x^2$ …(B)

We get, $\tan\theta = 2$

$\therefore \quad \cos\theta = \dfrac{1}{\sqrt{5}}$

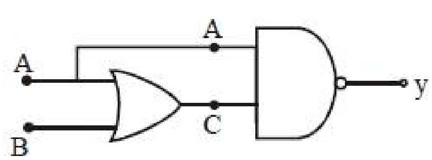
Also, $\dfrac{g}{2u^2\cos^2\theta} = 9$

$\Rightarrow \quad \dfrac{10}{2 \times 9 \times \left(\frac{1}{\sqrt{5}}\right)^2} = u^2 \quad ; \quad u^2 = \dfrac{25}{9}$

$\Rightarrow \quad u = \dfrac{5}{3} \text{m/s}$

25. $B = \dfrac{\mu_0 i}{2R} = \dfrac{\mu_0 q\omega}{2R\, 2\pi}$

$\Rightarrow \quad q = 3 \times 10^{-5}$ C

26.


C = A + B

and $y = \overline{A.C}$

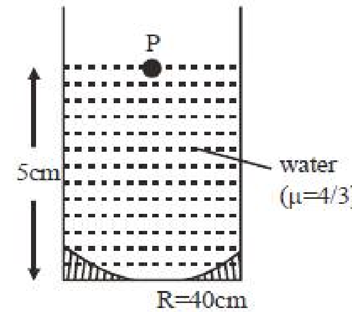
A	B	C = (A + B)	A.C.	$y = \overline{A.C}$
0	0	0	0	1
0	1	1	0	1
1	0	1	1	0
1	1	1	1	0

27.

	ΔE	ΔW	ΔQ
ab			250
bc		0	60
ca	–180		

	ΔE	ΔW	ΔQ
ab	120	130	250
bc	60	0	60
ca	–180		

28. Light incident from particle P will be reflected at mirror.



$$u = -5\text{cm},\ f = m - \frac{R}{2} = -20 \text{ cm}$$

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f} \quad ; \quad \boxed{v_1 = +\frac{20}{3}\text{cm}}$$

This image will act as object for light getting refracted at water surface.

So, object distance $d = 5 + \frac{20}{3} = \frac{35}{3}\text{cm}$

Below water surface.

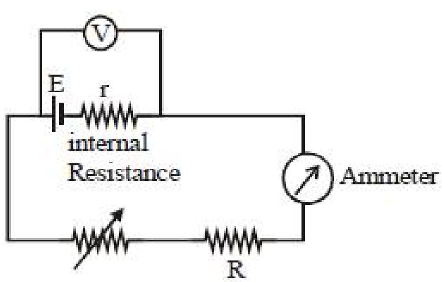
After refraction, final image is at

$$d' = d\left(\frac{\mu_2}{\mu_1}\right) = \left(\frac{35}{3}\right)\left(\frac{1}{4/3}\right)$$

$$= \frac{35}{4} = 8.75 \text{ cm}$$

$$\approx 8.8 \text{ cm}$$

29.



V = E – Ir

When $V = V_0 = 0 \Rightarrow 0 = E - Ir$

$\therefore$ E = r

When I = 0, V = E = 1.5 V

$\therefore$ r = 1.5 Ω

30. Angular momentum conservation

$MV_0L = MV_1(L - \ell)$

$$V_1 = V_0\left(\frac{L}{L-\ell}\right)$$

$w_g + w_p = \Delta KE$

$$-mg\ell + w_p = \frac{1}{2}m\left(V_1^2 - V_0^2\right)$$

$$w_p = mg\ell + \frac{1}{2}mV_0^2\left(\left(\frac{L}{L-\ell}\right)^2 - 1\right)$$

$$= mg\ell + \frac{1}{2}mV_0^2\left(\left(1-\frac{L}{L-\ell}\right)^{-2} - 1\right)$$

Now, $\ell << L$

By, Binomial approximation

$$= mg\ell + \frac{1}{2}mV_0^2\left(\left(1+\frac{L}{L-\ell}\right)^{-2} - 1\right)$$

$$= mg\ell + \frac{1}{2}mV_0^2\left(\frac{2\ell}{L}\right)$$

$$W_P = mg\ell + mV_0^2\frac{\ell}{L}$$

Here, V_0 = maximum velocity = $\omega \times A = \left(\sqrt{\frac{g}{L}}\right)(\theta_0 L)$

So, $w_p = mg\ell + m\left(\theta_0\sqrt{gL}\right)^2\frac{\ell}{L}$

$$= \; = mg\ell\left(1+\theta_0^2\right)$$

PART B – CHEMISTRY

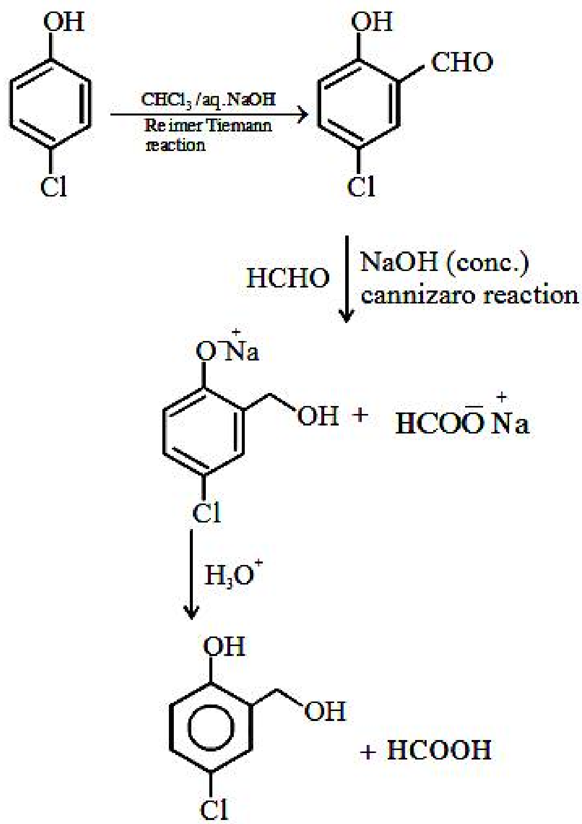
31. $X_{solvent} = 0.8 = 8/10$

$N_{Total} = 10$, $n_{solutent} = 8$, $n_{solute} = 2$

Wt of solvent = 8×18

$$\text{Molality} = \frac{2\times 1000}{8\times 18}$$

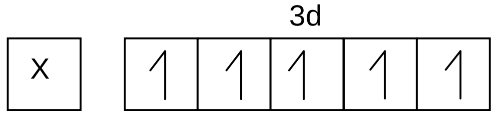
32.



33. pπ-dπ bonding in $N(SiH_3)_3$.

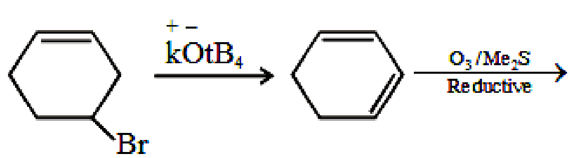
34. When Fe^{2+} oxidizes to Fe^{3+}

3d



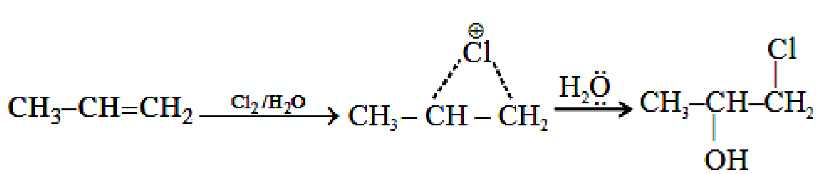
35. Smog photochemical is NO, NO_2, O_3.

36.



$CHO–CH_2–CH_2–CHO + OHC–CHO$

37.



38. Smaller the size of cation, more polarisability high thermal decomposition of carbonates.

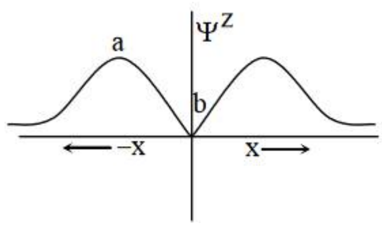
39. $15 \rightarrow 1s^2, 2s^2, 2p^6, 3s^2, 3p^3 \}$ Period- III, Group-V

40. Peptization is process to form collidal solution by adding peptizing agent.

41. $Zn \xrightarrow{HCl} ZnCl_2 + H_2$

$Zn \xrightarrow{NaOH} Na_2ZnO_2 + H_2$

42.



At a & c, probability of finding electron is maximum

43. $2Cu^{\oplus} \longrightarrow Cu^{+2} + Cu$

44. Mol. wt is of 1 mol

AB_2 A + 2B = 25

A_2B_2 2A + 2B = 30

45. Basic unit is silicate $(SiO_4)^{4-}$

46. $\Delta C_P = \dfrac{\Delta H_2 - \Delta H_1}{T_2 - T_1}$

47. Thermosetting are cross-linked polymer.

48. Two nearest tetrahedral voids are along one body-diagonal.

49. Canary yellow ppt comes in test of PO_4^{3-} ion.

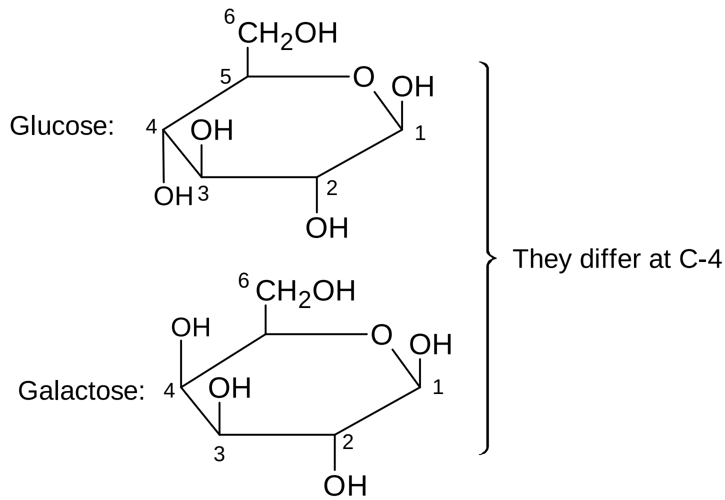
50. Concentration(on dressing), i.e. washing of ore.

51. $-\dfrac{1}{x}\left(\dfrac{dA}{dt}\right) = \dfrac{1}{y}\left(\dfrac{dB}{dt}\right)$

$\log_{10}\left[-\dfrac{d(A)}{dt}\right] = \left[\log_{10}\left(\dfrac{dB}{dt}\right)\right] + \log\dfrac{x}{y}$ $]\log\dfrac{x}{y} = \log 0.3$

$\left(\dfrac{x}{y} = 2\right)$

52.



53. Not always double stranded α-helix.

54. $Al(OH)_3 \rightleftharpoons Al^{3+} + 3OH^-$

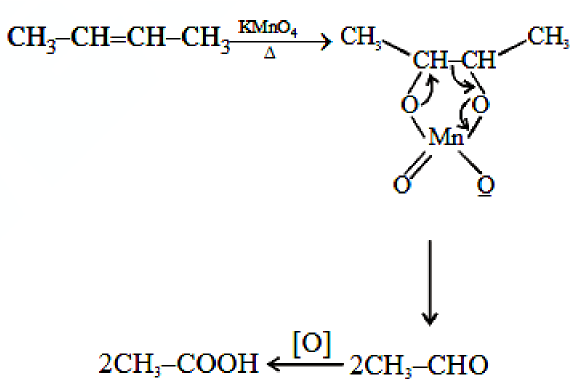
$\qquad\qquad s \qquad (3s + 0.2)$

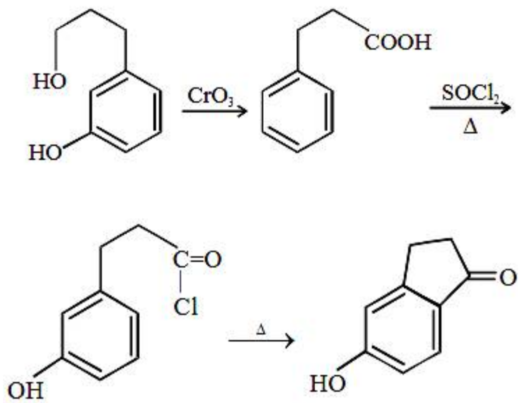
$2.4 \times 10^{-24} = s(3s + 0.2)^3$

55. CH_3O-, CH_3-, H is electron donating, But $-NO_2$ is electron withdrawing group.

56. Lower the standard reduction potential, more the ability to get reduced higher the oxidizing power.

57.



58. 

59. Ligand filed exert mass repulsion along x, y axis as compared to Z-axis so $d_{x^2-y^2}$ and d_{xx} will have increase in energy y

60. $W = - P_{ext}(V_2 – V_1)$

PART C – MATHEMATICS

61. Tangents $y^2 = 12x \Rightarrow y = 2x + \frac{3}{m}$

$\frac{x^2}{1} - \frac{y^2}{8} = 1 \Rightarrow y = mx \pm \sqrt{m^2 - 8}$

Common tangent gives

$\therefore \frac{3}{m = \pm\sqrt{m^2 - 8}}$

$m^4 - 8m^2 - 9 = 0$

$m = \pm 3$

$\therefore y = 3x + 1 \quad P\left(-\frac{1}{3}, 0\right)$

$y = -3x - 1$

P divides SS’ in 5 : 4.

$\frac{x^2}{1} - \frac{y^2}{8} = 1$

$e = 3$

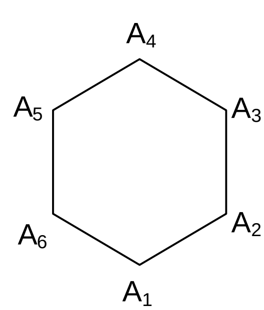
$ae = 3$

$S = (3,0)$

$S' = (-3, 0)$

62. Only two equilateral triangle are possible $A_1A_3A_5$ and $A_2A_5A_6$

$\frac{2}{{}^6C_3} = \frac{2}{20} = \frac{1}{10}$



63. $\lim_{x\to 2}\frac{\int_{6}^{f(x)} 4t^3 dt}{x-2}$

$=\lim_{x\to 2}\frac{4.f^3(x).f'(x)}{1}$

$=4f^3(2)f'(2)=18$

64. $\int\frac{2x^3-1}{x^4+x}dx$

$\int\frac{2x-\frac{1}{x^2}}{x^2+\frac{1}{x}}dx$

$x^2+\frac{1}{x}=t$

$\left(2x-\frac{1}{x^2}\right)dx=dt$

$\int\frac{dt}{t}=\ell n(t)+C$

$=\ell n\left(x^2+\frac{1}{x}\right)+C$

65. $(1+x)(1-x)^{10}(1+x+x^2)^9$

$(1-x^2)(1-x^3)^9$

${}^9C_6=84$

66. Since ${}^{21}C_0+.....+{}^{21}C_{10}+{}^{21}C_{11}+.......+{}^{21}C_{21}=2^{21}$

$\Rightarrow$ but we have to select 10 objects and ${}^{21}C_0+.....+{}^{21}C_{10}={}^{21}C_{11}+.......+{}^{21}C_{21}$

$\left({}^{21}C_0+.....+{}^{21}C_{10}\right)=2^{20}$

67. $np=8$

$npq=4$

$q=\frac{1}{2}\Rightarrow p=\frac{1}{2}$

$n=16$

$p(x=r)={}^{16}C_r\left(\frac{1}{2}\right)^{16}$

$p(x\le 2)=\frac{{}^{16}C_0+{}^{16}C_1+{}^{16}C_2}{2^{16}}$

$=\frac{137}{2^{16}}$

68. $|z-i| = |z-1|$

gives $y = x$

69. $f(x) = x\sqrt{kx - x^2}$

$f'(x) = \dfrac{3kx - 4x^2}{2\sqrt{kx - x^2}}$

For increasing $f(x) \geq 0$

$kx - x^2 \geq 0$

$x^2 - kx \leq 0$

$x(x-k) \leq 0$ so $x \in [0, 3)$

$+ve$ $\boxed{x \geq 3}$

$3kx - 4x^2 \geq 0$

$4x^2 - 3kx \leq 0$

$4x\left(x - \dfrac{3k}{4}\right) \leq 0$

$3 - \dfrac{3k}{4} \leq 0$

$k \geq 4$

minimum value of k is $m = 4$

$f(x) = x\sqrt{kx - x^2}$

$= 3\sqrt{4 \times 3 - 3^2} = 3\sqrt{3}, M = 3\sqrt{3}$

70. $x_1 + ... + x_4 = 44$

$x_5 + ... + x_{10} = 96$

$\bar{x} = 14, \sum x_i = 140$

Variance $= \dfrac{\sum x_i^2}{n} - \bar{x}^2 = 4$

Standard deviation = 2

71. $\{(x,y) : y^2 \leq 4x,\ x + y \leq 1,\ x \geq 0,\ y \geq 0\}$

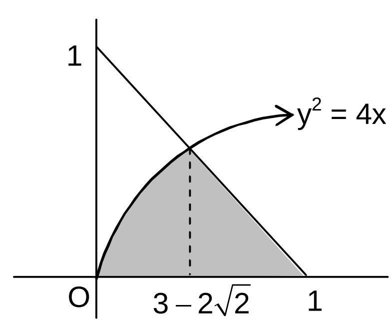
$A \int_0^{3-2\sqrt{2}} 2\sqrt{x}\,dx + \dfrac{1}{2}\left(1 - \left(3 - 2\sqrt{2}\right)\right)\left(1 - \left(3 - 2\sqrt{2}\right)\right)$

$= \dfrac{2\left[x^{3/2}\right]_0^{3-2\sqrt{2}}}{3/2} + \dfrac{1}{2}\left(2\sqrt{2} - 2\right)\left(2\sqrt{2} - 2\right)$

$= \dfrac{8\sqrt{2}}{3} + \left(-\dfrac{10}{3}\right)$

$a = \dfrac{8}{3},\ b = -\dfrac{10}{3}$

$a - b = 6$



72. $y^2 dx + x dy = \dfrac{dy}{y}$

$\dfrac{dx}{dy} + \dfrac{x}{y^2} = \dfrac{1}{y^3}$

$$IF = e^{\int \frac{1}{y^2} dy} = e^{-\frac{1}{y}}$$

$$e^{\frac{1}{y}}.x = \int e^{\frac{1}{y}}.\frac{1}{y^3} dy + C$$

$$xe^{\frac{1}{y}} = e^{\frac{1}{y}} + \frac{e^{-\frac{1}{y}}}{y} + C$$

$$C = -\frac{1}{e}$$

$$x = \frac{3}{2} - \frac{1}{\sqrt{e}} \text{ when } y = 2$$

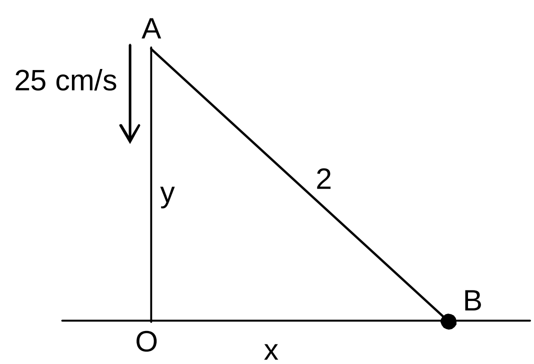
73. $375x^2 - 25x - 2 = 0$

$$\alpha + \beta = \frac{25}{375}, \alpha\beta = \frac{-2}{375}$$

$$\Rightarrow \left(\alpha + \alpha^2 + \text{...upto infinite terms}\right) + \left(\beta + \beta^2 + \text{..........upto infinite terms}\right)$$

$$= \frac{\alpha}{1-\alpha} + \frac{\beta}{1-\beta} = \frac{1}{12}$$

74. $x^2 + y^2 = 4 \left(\frac{dy}{dt} = -25\right)$



$$x\frac{dx}{dt} + y\frac{dy}{dt} = 0$$

$$\sqrt{3}\frac{dx}{dt} - 1(25) = 0$$

$$\frac{dx}{dt} = \frac{25}{\sqrt{3}} \text{cm / sec}$$

75. $1 + \sin^4 x = \cos^2 3x$

$\sin x = 0$ and $\cos 3x = 1$

$0, 2\pi, -2\pi, -\pi, \pi$

76. $e^y = xy = e$ differentiate w.r.t. x

$$e^y \frac{dy}{dx} + x\frac{dy}{dx} + y = 0$$

$$e^y \frac{dy}{dx} + x\frac{dy}{dx} + y = 0$$

$$\frac{dy}{dx}\left(x + e^y\right) = -y, \left.\frac{dy}{dx}\right|_{(0,1)} = -\frac{1}{e} \text{again differentiate w.r.t.x}$$

$$e^y.\frac{d^2y}{dx^2} + \frac{dy}{dx}.e^y.\frac{dy}{dx} + x.\frac{d^2y}{dx^2} + \frac{dy}{dx} + \frac{dy}{dx} = 0$$

$$\left(x + e^y\right)\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2.e^y + 2\frac{dy}{dx} = 0$$

$$e\frac{d^2y}{dx^2}+\frac{1}{e^1}e+2\left(-\frac{1}{e}\right)=0$$

$$\therefore \frac{d^2y}{dx^2}=\frac{1}{e^2}$$

77. $\sin^{-1}\left(\frac{12}{13}\right)-\sin^{-1}\left(\frac{3}{5}\right)$

$\sin^{-1}\left(x\sqrt{1-y^2}-y\sqrt{1-x^2}\right)$

$=\sin^{1}\left(\frac{33}{65}\right)=\cos^{-1}\left(\frac{56}{65}\right)=\frac{\pi}{2}-\sin^{1}\left(\frac{56}{65}\right)$

78. $3x^2+4y^2=12$

$\frac{x^2}{4}+\frac{y^2}{3}=1$

$x=2\cos\theta,\ y=\sqrt{3}\sin\theta$

Equation of normal is $\frac{a^2x}{x_1}-\frac{b^2y}{y_1}=a^2-b^2$

$2x\sin\theta-\sqrt{3}\cos\theta y=\sin\theta\cos\theta$

Slope $\frac{2}{\sqrt{3}}\tan\theta=-2 \quad \therefore \tan\theta=-\sqrt{3}$

Equation of tangent is it passes through (4, 4)

$3x\cos\theta+2\sqrt{3}\sin\theta y=6$

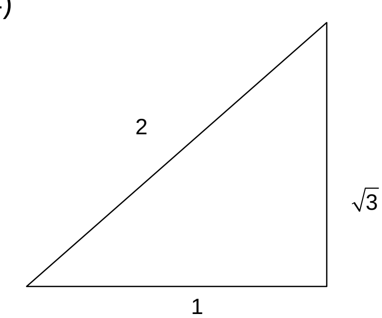
$12\cos\theta+8\sqrt{3}\sin\theta=6$

$\cos\theta=-\frac{1}{2},\sin\theta=\frac{\sqrt{3}}{2}\ \therefore\theta=120^o$

Hence point P is $\left(2\cos120^o,\ \sqrt{3}\sin120^o\right)$

$P\left(-1,\frac{2}{2}\right),\ Q(4,4)$

$PQ=\frac{5\sqrt{5}}{2}$



79. $\left(\vec{a}+\vec{b}\right)\times\left(\vec{a}-\vec{b}\right)$

$=2\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & 2 & -2\\ 3 & 2 & 2\end{vmatrix}$

$=2\left(8\hat{i}-8\hat{j}+4\hat{k}\right)$

Required vector $=\pm12\frac{\left(2\hat{i}-2\hat{j}-\hat{k}\right)}{3}$

$=\pm4\left(2\hat{i}-2\hat{j}-\hat{k}\right)$

80. $2y = 2\sin x \sin(x+2) - 2\sin^2(x+1)$

$2y = \cos 2 - \cos(2x+2) - (1-\cos(2x+2))$

$= \cos 2 - 1$

$2y = -2\sin^2 \frac{1}{2}$

$y = -\sin^2 \frac{1}{2} \le 0$

81. $f(x) = \sqrt{x},\ g(x) = \tan x, h(x) = \frac{1-x^2}{1+x^2}$

$fog(x) = \sqrt{\tan x}$

$hofog(x) = h\left(\sqrt{\tan x}\right) = \frac{1-\tan x}{1+\tan x}$

$= -\tan\left(\frac{\pi}{4} - x\right)$

$\phi(x) = \tan\left(\frac{\pi}{4} - x\right)$

$\phi\left(\frac{\pi}{3}\right) = \tan\left(\frac{\pi}{4} - \frac{\pi}{3}\right) = \tan\left(-\frac{\pi}{12}\right) = -\tan\frac{\pi}{12}$

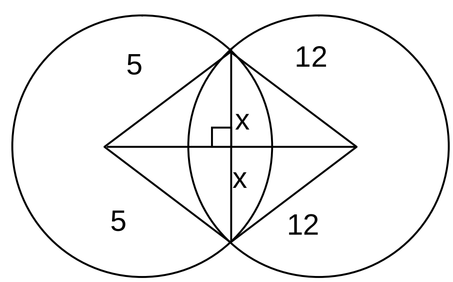
$= \tan\left(\pi - \frac{\pi}{12}\right) = \tan\frac{11\pi}{12}$

82. Let length of common chord $= 2x$

$\sqrt{25 - x^2} + \sqrt{144 - x^2} = 13$

After solving $x = \frac{12 \times 5}{13}$

$2x = \frac{120}{13}$



83. $\int_0^{\pi/2} \frac{\cot x \, dx}{\cot x + \operatorname{cosec} x}$

$\int_0^{\pi/2} \frac{\cos x}{\cos x + 1} = \int \frac{2\cos^2 \frac{x}{2} - 1}{2\cos^2 \frac{x}{2}}$

$\int_0^{\pi/2} \left(1 - \frac{1}{2}\sec^2 \frac{x}{2}\right) dx$

$\left[x - \tan\frac{x}{2}\right]_0^{\frac{\pi}{2}}$

$\frac{1}{2}[\pi - 2]$ $\quad m = \frac{1}{2}, n = -2$

$mn = -1$

84. $2\{2a + 3d\} = 16$

$3(2a + 5d) = -48$

$2a + 3d = 8$

$2a + 5d = -16$

$d = -12$

$S_{10} = 5\{44 - 9 \times 12\}$

$= -320$

85. $|B| = 5(-5) - 2\alpha(-\alpha) - 2\alpha$

$= 2\alpha^2 - 2\alpha - 25$

$1 + |A| = 0$

$\alpha^2 - \alpha - 12 = 0$

Sum = 1

86. $\frac{x-2}{3} = \frac{y+1}{2} = \frac{z-1}{-1} = \lambda$

$x = 3\lambda + 2, y = 2\lambda - 1, z = -\lambda + 1$

Intersection with plane $2x + 3y - z + 13 = 0$

$2(3\lambda + 2) + 3(2\lambda - 1) - (-\lambda + 1) + 13 = 0$

$13\lambda + 13 = 0$

$\lambda = -1$

$\therefore P(-1, -3, 2)$

Intersection with plane

$3x + y + 4z = 16$

$3(3\lambda + 2) + (2\lambda - 1) + 4(-\lambda + 1) = 16$

$\lambda = 1$

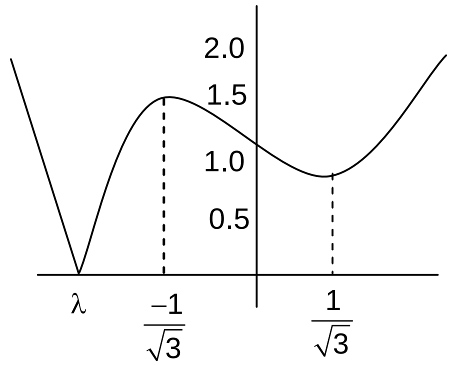
$Q(5, 1, 0)$

$PQ = \sqrt{6^2 + 4^2 + 2^2} = \sqrt{56} = 2\sqrt{14}$

87. Volume of paralleopiped $= \left\| \begin{matrix} 1 & \lambda & 1 \\ 0 & 1 & \lambda \\ \lambda & 0 & 1 \end{matrix} \right\|$

$f(\lambda) = |\lambda^3 - \lambda + 1|$

Its graphs as follows



where $\lambda \approx -1.32$

For minimum value of volume of paralelopiped and corresponding value of λ; the minimum value is zero, $\because$ cubic always has at least one real root.

Hence answer to the question must be root of cubic $\lambda^3 - \lambda + 1 = 0$. None of the options satisfies the cubic.

Hence Question must be Bonus.

88. $A = A', B = -B'$

$$A + B = \begin{bmatrix} 2 & 3 \\ 5 & -1 \end{bmatrix} \quad \text{..........(1)}$$

$$A' - B' = \begin{bmatrix} 2 & 5 \\ 3 & -1 \end{bmatrix}$$

$$A - B = \begin{bmatrix} 2 & 5 \\ 3 & -1 \end{bmatrix} \quad \text{..........(2)}$$

After adding equation (1) and (2)

$$A = \begin{bmatrix} 2 & 4 \\ 4 & -1 \end{bmatrix}, B = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

$$AB = \begin{bmatrix} 4 & -2 \\ -1 & -4 \end{bmatrix}$$

89. $P \to (\sim q \vee r)$

$\sim p \vee (\sim q \vee r)$

$$\left.\begin{matrix} \sim p \to F \\ \sim q \to F \\ \sim r \to F \end{matrix}\right\} \Rightarrow \left.\begin{matrix} p \to T \\ q \to T \\ r \to F \end{matrix}\right\}$$

90. $$\underbrace{\left[-\frac{1}{3}\right] + \left[-\frac{1}{3} - \frac{1}{100}\right] + ... + \left[\frac{1}{3} - \frac{66}{100}\right]}_{(-1)67} + \underbrace{\left[-\frac{1}{3} - \frac{67}{100}\right] + ... + \left[-\frac{1}{3} - \frac{99}{100}\right]}_{-2(33)} = -133$$